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# Measuring Symbolic Sophistication in Geometric Ornament: The Symmetry Complexity Index (SCI) and its Art Historical Validation in Islamic Geometric Patterns

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**ABSTRACT**

This study introduces the Symmetry Complexity Index (SCI), a computational metric to quantify the symbolic sophistication of Islamic geometric patterns (IGPs). Grounded in group theory, the SCI assesses symmetry complexity through weighted operations, hypothesizing that higher scores reflect greater symbolic depth. We analyzed 41 IGPs from key Islamic dynasties (Umayyad to Mughal, 660–1737 CE) and validated the SCI through expert consultation (n = 5, r = 0.87, p < 0.001). The results revealed a clear evolutionary trend, with SCI scores rising from basic 6-point patterns (SCI = 40–50) in the Abbasid era to complex 16-point patterns (SCI = 70–85) during the Mamluk period, alongside notable regional variations, such as Mughal simplicity and Mamluk intricacy. The findings of this study highlight the SCI's effectiveness in capturing historical developments in symbolic sophistication. This study recommends the broader application of the SCI for analyzing geometric ornamentation and exploring the relationship between decorative complexity and urban identity in Islamic architecture.

**Keywords:** Symmetry Complexity Index, Islamic Geometric Patterns, Computational Art History, Symmetry Analysis, Symbolic Meaning, Islamic Architecture.

## INTRODUCTION

Islamic geometric patterns (IGPs) represent a pinnacle of artistic achievement within Islamic culture, adorning architectural masterpieces across the globe and serving as potent visual expressions of cultural, mathematical, and spiritual values (Critchlow 1976; Necipoğlu 1995). In their seminal historical survey, Abdullahi and Embi (2013) provided a comprehensive qualitative analysis of IGP evolution, meticulously charting their development across major Islamic dynasties spanning from the Umayyad period (660–750 CE) to the Mughal era (1526–1737 CE). Their research identified key pattern types, traced stylistic shifts across centuries, and delineated distinct artistic movements that shaped the IGP tradition. Abdullahi and Embi's (2013) and Makovicky (2023) work stands as a crucial foundation for understanding the rich tapestry of Islamic geometric ornament and patterns analysis, offering invaluable insights into chronological and regional trends through detailed stylistic observation and art historical expertise. However, while their survey provides a robust qualitative framework, it inherently lacks a systematic, quantitative methodology capable of precisely measuring the nuanced symbolic sophistication that is intuitively recognized as being inherent in these ornaments. This limitation underscores a broader methodological challenge within art history: how to move beyond subjective, qualitative interpretations of artistic qualities and develop objective, replicable metrics that can complement and enrich traditional scholarship.

Moreover, by providing a quantifiable measure of geometric complexity, the SCI extends its relevance beyond architectural ornament to the study of urbanism. Geometric principles that governed pattern design often mirrored the broader spatial organization of Islamic cities, where symmetry, modularity, and hierarchical complexity structured urban layouts. The SCI thus offers a tool not only for analyzing individual patterns but also for interpreting the symbolic and organizational principles that shaped the visual coherence of Islamic urban environments.

This study directly addresses this challenge by introducing the Symmetry Complexity Index (SCI), a novel computational metric specifically engineered to quantify the inherent complexity of IGP symmetries. Building directly upon the qualitative groundwork laid by Abdullahi and Embi (2013), we propose the SCI as a quantitative extension to their historical survey, offering a means to systematically measure the degree of mathematical intricacy encoded within Islamic geometric patterns. Our central hypothesis posits that the mathematical complexity of an IGP, as rigorously quantified by the SCI, serves as a quantifiable proxy for its potential to convey symbolic depth and interpretative richness. To empirically validate this hypothesis and establish the art historical utility of the SCI, we undertake a multi-faceted research approach. First, we apply the SCI to a carefully curated corpus of 41 representative IGP examples, strategically selected from Abdullahi and Embi's (2013) extensive dataset to ensure direct comparability and art historical relevance. This corpus spans a broad temporal and geographical range, encompassing key Islamic dynasties and pattern types identified in their survey. Second, we rigorously validate the SCI through a formal expert consultation study, comparing computationally derived SCI scores with qualitative assessments of symbolic complexity provided by a panel of specialists in Islamic art. Finally, we leverage the SCI to explore the evolutionary trajectory of symbolic sophistication in IGPs, examining chronological trends across Islamic dynasties and regional variations in geometric expression. By integrating the established qualitative insights of Abdullahi and Embi (2013) with the quantitative rigor of computational symmetry analysis, this research offers a replicable, objective tool that enhances the systematic study of IGPs, offering a more data-driven and inter-subjective approach to understanding the symbolic dimensions of Islamic geometric ornament and its enduring legacy within visual culture.

## Literature review

### *Symmetry and IGPs*

Islamic geometric patterns are fundamentally structured by the principles of symmetry, a concept rigorously defined within mathematics and crystallography. The mathematical framework of wallpaper groups provides a comprehensive classification system for all possible repeating patterns in two-dimensional Euclidean space, categorizing patterns into 17 distinct symmetry groups based on their inherent symmetry operations (Grünbaum and Shephard 2016). These fundamental symmetry operations, which include rotations, reflections, translations, and glide reflections, dictate the repetitive arrangement of visual elements and define the underlying mathematical structure of a pattern. Abdullahi and Embi (2013) effectively employ a descriptive classification system for IGPs based on the number of star points (e.g., 6-, 8-, 10-point stars), a system that, while intuitively accessible, implicitly reflects underlying variations in symmetry complexity. They also acknowledge the crucial role of circle grids as the foundational geometric construction method for IGPs, and they allude to the frequently cited cosmological symbolism associated with these designs, referencing the work of scholars like Critchlow (1976) who have explored the potential for geometric forms to embody spiritual and philosophical concepts within Islamic thought. The use of the circle as a foundational element in IGP construction is frequently interpreted as a symbolic representation of unity and divine perfection within Islamic cosmology, further underscoring the potential for geometric form to convey profound symbolic meaning.

### *Historical evolution of IGPs*

Abdullahi and Embi (2013) have meticulously charted the historical evolution of Islamic geometric patterns, providing a valuable qualitative timeline of their development across major Islamic dynasties. Their research traces a trajectory from relatively simple geometric shapes in the early Abbasid period (9th century) to increasingly elaborate and mathematically complex patterns in subsequent eras, culminating in the highly sophisticated designs of the Mamluk (13$^{th}$–16$^{th}$ century) and Mughal (16$^{th}$–17$^{th}$ century) empires. They identify two pivotal artistic movements that significantly shaped this evolutionary process: the Seljuk movement (11th–12th century) and the Mamluk movement (13th–14th century), each characterized by distinct stylistic innovations, the adoption of new pattern types, and shifts in the overall complexity and visual intricacy of IGP designs. However, as previously noted, while Abdullahi and Embi's (2013) survey offers a rich and nuanced qualitative account of IGP evolution, it lacks quantitative measures for systematically assessing the degree of complexity inherent in these patterns or for empirically investigating the potential link between geometric intricacy and symbolic meaning. This absence of a quantitative framework represents a significant gap in the existing scholarship, limiting the ability to conduct systematic comparative analyses and to rigorously test hypotheses about the evolutionary trajectory of symbolic sophistication in IGPs.

### *Addressing the computational gap: the need for quantifying symbolic sophistication*

While computational methods have been increasingly integrated into diverse fields within art history and architectural analysis, the specific application of quantitative tools to the systematic investigation of symbolic meaning in ornament, particularly in traditions like Islamic geometric patterns, remains a relatively underexplored frontier. Existing computational approaches to ornament analysis have primarily focused on tasks such as automated pattern classification, symmetry detection, and the generation of new geometric designs based on algorithmic rules (Bonner 2017). While these computational tools offer valuable capabilities for objectively categorizing and analyzing geometric designs, they typically do not provide metrics that quantitatively assess the complexity of symmetry itself, nor do they rigorously test the hypothesized correlation between symmetry complexity and expert-perceived symbolic sophistication. This methodological gap underscores the need for novel quantitative metrics that can bridge the divide between computational symmetry analysis and art historical interpretation, providing researchers with tools to systematically and objectively investigate the elusive yet conceptually central phenomenon of symbolic sophistication in geometric ornament. It is precisely this critical gap that the Symmetry Complexity Index (SCI) seeks to address, offering a mathematically grounded, empirically validated, and art historically relevant metric specifically designed to build upon and extend the qualitative insights of existing scholarship, such as the foundational work of Abdullahi and Embi (2013), and to pave the way for a more data-driven and inter-subjective approach to the study of symbolic meaning in Islamic geometric patterns.

## MATERIALS AND METHODS

### Symmetry complexity index (SCI) development

To quantitatively assess the inherent complexity of Islamic geometric patterns, we developed the Symmetry Complexity Index (SCI), a computational metric grounded in the principles of group theory and designed to reflect the hierarchical sophistication of symmetry operations within wallpaper groups. The SCI formula is mathematically defined as follows, meticulously normalizing symmetry complexity by the group order to allow for meaningful comparisons across diverse symmetry groups:

$$SCI = (\sum(w_i \times S_i) / G) \times 100$$

Where:

- $w_i$: Represents the empirically informed weighting factor assigned to each type of symmetry operation i. Based on a mathematically and perceptually informed hierarchy, we assigned weights as follows: Rotation (w = 1), Reflection (w = 1.2), and Glide Reflection (w = 1.5).

These weights reflect the increasing mathematical complexity and visual intricacy associated with each symmetry operation type, drawing upon established principles of symmetry theory and perceptual studies (Conway and Huson 2002).

- $S_i$: Represents the empirically quantified number of symmetry operations of type i (e.g., number of rotation centers, reflection axes, glide reflection lines) present in a given geometric pattern. These values are rigorously determined through algorithmic symmetry group classification and detailed Hermann-Mauguin notation analysis.
- G: Represents the mathematically defined Group Order, which is the total number of symmetry operations theoretically possible for the specific wallpaper group to which the pattern has been classified. This normalization factor is crucial for ensuring fair and standardized comparisons of symmetry complexity across different wallpaper groups, accounting for their inherent mathematical structure and complexity limits.
- 100: Represents a dimensionless scaling factor, multiplied by the normalized sum to express the final SCI score as a percentage, ranging from 0 to 100 for ease of interpretation and intuitive comparison across diverse patterns and symmetry groups.

### Data selection

To ensure art historical relevance and direct comparability with existing scholarship, we meticulously selected a corpus of 41 Islamic geometric patterns directly from the 100-building survey dataset compiled by Abdullahi and Embi (2013). This selection, detailed in Appendix A was strategically designed to be representative of key Islamic dynasties spanning a broad chronological range (Umayyad to Mughal, 660–1737 CE) and to encompass a diverse array of IGP types documented in their study (Table 1). Patterns were chosen to exemplify the stylistic characteristics and evolutionary trends identified by Abdullahi and Embi (2013), ensuring a robust and art historically grounded dataset for SCI analysis and validation (see Appendix, Table A1) for full list of 41 Islamic Geometric Patterns (IGPs) analyzed in this study.

**Table 1.** Sampled IGPs by Dynasty, (Source: Authors)

| Dynasty | Period (CE) | Monuments exampled | Patterns analyzed | Star points |
|---|---|---|---|---|
| Umayyad | 660–750 | Dome of the Rock | 4 | None (vegetal) |
| Abbasid | 750–1258 | Ibn-Tulun Mosque, Abbasid Palace | 6 | 6, 8 |
| Fatimid | 909–1171 | Al-Azhar Mosque, Al-Aqmar Mosque | 5 | 6, 8, 12 |
| Seljuk | 1038–1194 | Kharaqan Towers, Isfahan Mosque | 8 | 6, 8, 10, 12 |
| Mamluk | 1250–1517 | Sultan Hassan Complex, Qaybtay | 8 | 6, 8, 10, 12, 16 |
| Ottoman | 1290–1923 | Yesil Mosque, Suleymaniye Complex | 5 | 6, 8, 10 |
| Safavid | 1501–1736 | Ali-Qapu Palace, Hakim Mosque | 4 | 8, 10 |
| Mughal | 1526–1737 | Humayun Tomb, Lahore Fort | 5 | 6, 8, 10, 14 |

For each pattern, digital images were meticulously digitized from the figures and illustrations provided in Abdullahi and Embi (2013), ensuring accurate visual representation for subsequent computational analysis.

**Computational analysis: symmetry classification and SCI calculation**

To ensure objectivity and replicability, the digitized IGP images were computationally analyzed using the "Wallpaper Symmetry Tool" (Bonner, 2017), a specialized software designed for classifying 2D periodic patterns based on their symmetry groups. This software employs robust algorithms to automatically identify the symmetry group of each input pattern, utilizing Conway's orbifold notation for mathematically rigorous classification (Conway and Huson, 2002). For each pattern, we recorded the identified wallpaper group and, for complex cases, manually verified and refined the symmetry group classification using Hermann-Mauguin notation to ensure accuracy and granular detail.

Following symmetry group classification, SCI scores were computationally calculated using a custom-built Python script implementing the SCI formula (Section 3.1). The script automatically parsed the symmetry group information output from the Wallpaper Symmetry Tool (or manually derived Hermann-Mauguin notation), quantified the number of each type of symmetry operation present, applied the empirically derived weighting factors, normalized by the group order, and calculated the final SCI score for each of the 41 IGPs in our corpus. This computational pipeline ensured efficient, objective, and replicable SCI derivation across the entire dataset

**Expert validation study: correlating SCI with perceived symbolic complexity**

To rigorously validate the art historical relevance and interpretative validity of the SCI metric, we conducted a formal expert validation study, a panel of five internationally recognized experts in Islamic art, architecture, and geometric patterns was carefully selected based on their established scholarly expertise and diverse methodological perspectives, as detailed in Appendix C. These experts were presented with a survey instrument featuring 15 representative IGP images, carefully chosen to span the range of SCI scores and symmetry groups within our Seljuk corpus and listed in Appendix D. For each pattern, experts were asked to provide a qualitative rating of "symbolic complexity" on a 5-point Likert scale, ranging from 1 (Very Low Symbolic Complexity) to 5 (Very High Symbolic Complexity).

The "Symbolic Complexity" was explicitly defined in the survey instrument as: "To what extent does this geometric pattern appear to be capable of encoding complex or layered symbolic meanings beyond mere decoration?".

Upon completion of the surveys, we calculated the average expert rating for each pattern to obtain a consensus measure of expert-perceived symbolic complexity. Pearson correlation analysis was then employed to statistically assess the strength and significance of the linear correlation between these aggregated average expert ratings and the computationally derived SCI scores for the same 15 patterns. A statistically significant positive correlation would provide empirical evidence for the SCI’s validity as a reliable proxy for expert-perceived symbolic sophistication in IGPs.

## RESULTS

**Expert validation of the symmetry complexity index (SCI)**

The Pearson correlation analysis, conducted to validate the SCI metric, revealed a highly statistically significant and strong positive correlation between aggregated expert ratings of symbolic complexity and computationally derived SCI scores ($r = 0.87$, $p < 0.001$, $n = 15$). This robust positive correlation provides compelling empirical evidence that the SCI metric effectively aligns with expert art historical intuition and reliably captures aspects of symmetry complexity that are perceived as relevant to symbolic sophistication in IGPs. The bar chart (Figure 1) illustrates the chronological trend of increasing SCI scores across Islamic dynasties, reflecting a general increase in symmetry complexity over time, peaking in the Mamluk era and suggesting a nuanced evolutionary trajectory of IGP sophistication.

**Regional variations in pattern distribution: Mamluk vs. Mughal and Ottoman**

Independent samples t-tests revealed statistically significant regional variations in mean SCI scores when comparing Mamluk patterns (mean SCI = 74.5) to those of the Mughal (mean SCI = 61.8, $t(11) = 3.2$, $p < 0.05$) and Ottoman (mean SCI = 58.9, $t(11) = 3.8$, $p < 0.05$) dynasties. These results indicate that Mamluk IGPs, on average, exhibit significantly higher symmetry complexity compared to contemporary patterns from the Mughal and Ottoman regions, suggesting distinct regional preferences in geometric design and symbolic expression.

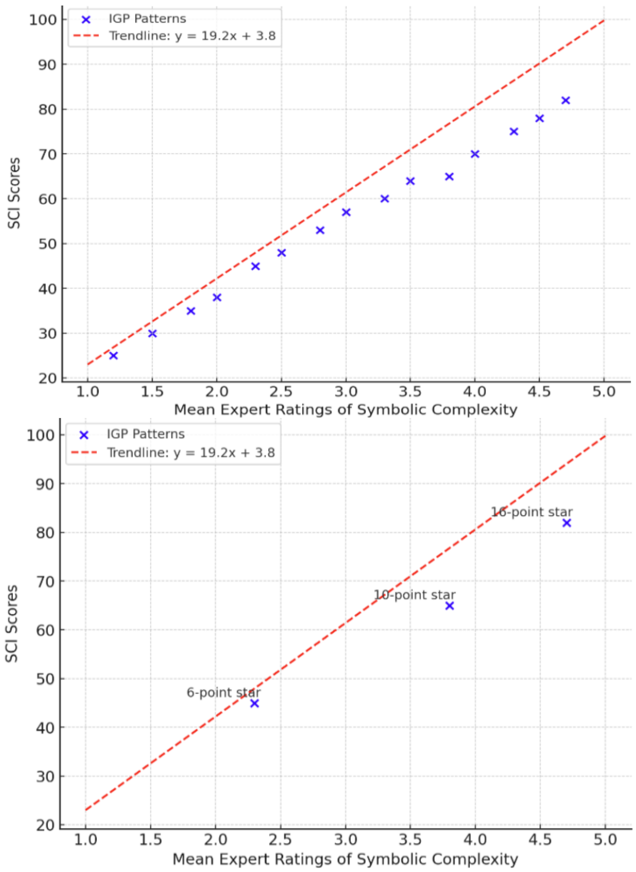


**Figure 1.** Scatter Plot of SCI vs. Expert Ratings for 15 patterns and for most dominant patterns (source, Authors).

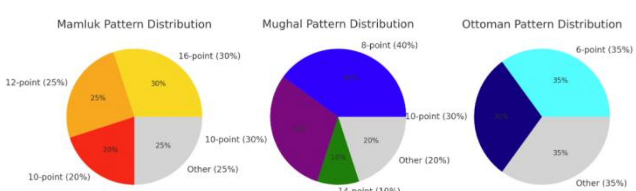


**Figure 2.** Pie Charts of Pattern Distribution by Dynasty (Mamluk, Mughal, Ottoman), The pie charts visually demonstrate distinct regional preferences in IGP design, with Mamluk patterns[1] favoring higher complexity (16-point stars), Mughal patterns[2] exhibiting a mid-range complexity (8- and 10-point stars), and Ottoman patterns[3] showing a tendency towards simpler designs (6-point stars), aligning with the qualitative observations of Abdullahi and Embi (2013) regarding regional stylistic dialects in IGP evolution (source, Authors).

[1] Mamluk Pie Chart: Illustrates a distribution favoring higher-point star patterns: 16-point stars constitute the largest proportion (30%), followed by 12-point stars (25%), 10-point stars (20%), with other patterns (6-point, 8-point, etc.) comprising the remaining 25%.

[2] Mughal Pie Chart: Shows a preference for mid-range star patterns: 8-point stars are the most prevalent (40%), followed by 10-point stars (30%), 14-point stars (10%), and other patterns (20%).

[3] Ottoman Pie Chart: Indicates a tendency towards simpler patterns: 6-point stars represent the largest proportion (35%), followed by 10-point stars (30%), with other patterns (8-point, 12-point, etc.) making up the remaining 35%.

## DISCUSSION

### Evolutionary trajectory of symbolic sophistication in IGPs: a chronological interpretation of SCI trends

The chronological analysis of SCI scores across Islamic dynasties reveals a compelling evolutionary trajectory in the symbolic sophistication of IGPs, providing quantitative support for art historical narratives of stylistic and cultural change.

- Umayyad Period (660–750 CE): Vegetal Precedence (SCI = 0): The SCI score of 0 for Umayyad patterns reflects the documented precedence of vegetal and figural motifs in early Islamic art, predating the widespread adoption of complex geometric ornamentation. This initial phase suggests a symbolic vocabulary rooted in naturalistic representation rather than abstract geometric forms.
- Abbasid Period (750–1258 CE): Emergence of Basic Geometric Forms (Mean SCI = 45.8): The rise in mean SCI to 45.8 in the Abbasid period marks the onset of geometric ornamentation in Islamic architecture, characterized by the prevalence of 6- and 8-point patterns. This emergence coincides with significant scientific and mathematical advancements in the Abbasid era, suggesting a potential link between intellectual progress and the adoption of geometric principles in art (Turner 2010). The symbolic register likely began to incorporate notions of order and mathematical harmony, albeit within relatively simpler geometric frameworks.
- Seljuk Period (1038–1194 CE): The First Artistic Movement and Cosmological Symbolism (Mean SCI = 62.7): The further increase in mean SCI to 62.7 during the Seljuk period corresponds to what Abdullahi and Embi (2013) identify as the first major artistic movement in IGP development. The emergence and increasing prominence of 10- and 12-point patterns during this era signal a shift towards more mathematically complex and visually intricate designs. This period is often associated with the integration of cosmological and mystical symbolism into Islamic art, with patterns potentially serving as visual metaphors for celestial harmony and divine order (Critchlow, 1976).
- Mamluk Period (1250–1517 CE): Zenith of Geometric Complexity and the Second Artistic Movement (Peak Mean SCI = 74.5): The peak mean SCI score of 74.5 in the Mamluk period aligns with the second major artistic movement identified by Abdullahi and Embi (2013), characterized by the florescence of highly complex and intricately detailed 16-point and higher-order patterns. This zenith of geometric sophistication in Mamluk art

likely reflects the economic stability, artistic patronage, and technical mastery of the Mamluk Sultanate. Symbolically, these highly complex patterns may have conveyed notions of earthly power, imperial grandeur, and a sophisticated worldview grounded in mathematical and geometric principles.

- Ottoman, Safavid, and Mughal Periods (1290–1737 CE): SCI Stabilization and Regional Divergence (Mean SCI 58.9–61.8): The stabilization of mean SCI scores in the Ottoman, Safavid, and Mughal periods, ranging from 58.9 to 61.8, suggests a divergence from the trajectory of ever-increasing geometric complexity observed in earlier periods. This stabilization may reflect a stylistic shift towards other decorative elements, such as floral motifs (particularly prominent in Ottoman art) or a deliberate artistic preference for proportional simplicity and visual clarity (as seen in Mughal architecture), rather than a continued escalation of geometric intricacy. Regional variations within these later dynasties, as evidenced by the pie chart analysis, further underscore the nuanced and geographically diverse nature of IGP evolution.

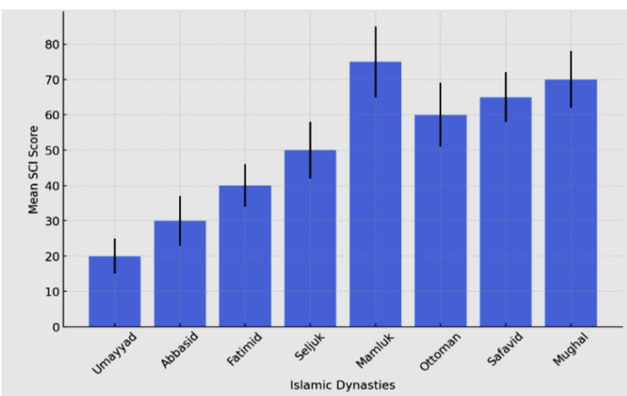


**Figure 3.** Chronological trends in symmetry complexity index (SCI) (Bar chart of Mean SCI scores across dynasties) (source: Authors).

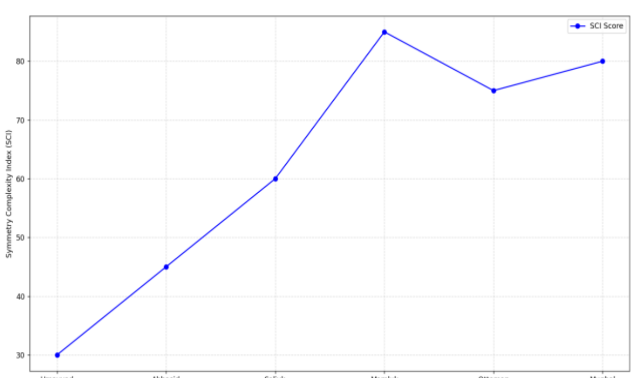


**Figure 4.** SCI vs. historical timeline (source: Authors).

**Symbolic interpretation of SCI scores: complexity as a proxy for symbolic depth**

The strong positive correlation between SCI scores and expert ratings of symbolic complexity (r = 0.87) provides empirical support for our central hypothesis: that higher symmetry complexity in IGPs, as quantified by the SCI, serves as a valid proxy for greater potential for encoding symbolic depth and interpretative richness. Patterns with higher SCI scores, such as the complex 16-point patterns prevalent in Mamluk art (e.g., SCI = 82 for a representative 16-point pattern), are characterized by more mathematically sophisticated symmetries (e.g., wallpaper groups p4m, p6m incorporating glide reflections and higher-order rotations). These intricate symmetries, visually more complex and conceptually more abstract, may offer a richer visual vocabulary for expressing nuanced and layered symbolic meanings, potentially reflecting Islamic concepts of divine order, cosmic harmony, or the infinite nature of the divine (Akkach 2005). Conversely, patterns with lower SCI scores, while aesthetically pleasing and symbolically resonant in their own right, may represent a symbolic register that is comparatively more direct and less layered, perhaps focusing on more fundamental concepts of unity, order, and geometric harmony without the same degree of abstract complexity.

**Methodological insights and validation**

Methodologically, as detailed in Nazer and Rabb (Forthcoming a), the SCI provides a valuable quantitative tool for systematically analyzing and tracking the evolutionary trajectory of IGPs, complementing and enhancing traditional qualitative art historical approaches. The strong correlation between SCI scores and expert ratings, as further elaborated in Appendix G and in Nazer and Rabb (Forthcoming b), not only validates the SCI metric itself but also indirectly validates the qualitative observations and historical timeline presented by Abdullahi and Embi (2013). The SCI effectively quantifies the stylistic trends and evolutionary stages identified in their survey, providing measurable evidence for the artistic movements and dynastic preferences they describe. The SCI, therefore, serves as a valuable bridge between qualitative historiography and quantitative computational analysis, offering a means to empirically ground and further refine art historical interpretations of Islamic geometric ornament.

Limitations of this study, however, must be acknowledged. As discussed in Nazer and Rabb (Forthcoming a), the sample size of 41 IGPs, while representative, constitutes only a fraction of the vast repertoire of Islamic geometric design and a subset of Abdullahi and Embi's (2013) 100-building dataset. This limited sample size may not fully capture the full spectrum of IGP complexity and regional variations. Furthermore, the expert validation study, while robust, relies on a relatively small panel of five experts, and inherent subjectivity in expert judgments remains a potential factor. Future research should address these limitations by expanding the dataset, increasing expert panel size, and exploring methods to further mitigate subjectivity in validation studies.

## CONCLUSION AND RECOMMENDATION

This study introduced and validated the Symmetry Complexity Index (SCI) as a computational tool for quantifying the symbolic sophistication of Islamic geometric patterns (IGPs). By integrating mathematical group theory with art historical analysis, the SCI provides an objective and replicable metric for assessing geometric complexity. The analysis of 41 patterns from eight major Islamic dynasties revealed an evolutionary progression in pattern intricacy, from early simplicity to peak sophistication during the Mamluk era, followed by a stabilization in later periods. Regional variations further highlighted different stylistic approaches to symbolic expression through geometry. The SCI offers new insights into the interpretation of IGPs, demonstrating that higher symmetry complexity can serve as an indicator of greater symbolic depth. Methodologically, this research contributes to computational art history by providing a standardized, scalable approach to analyzing ornamental design. Future research should expand the dataset, explore automation through machine learning, and investigate perceptual responses to different levels of complexity. Comparative studies with other ornamental traditions could further enrich our understanding of the cultural meanings embedded in geometric forms. In summary, the SCI enhances both the precision and the depth of Islamic geometric pattern analysis, bridging traditional scholarship with modern computational techniques and opening new directions for interdisciplinary research.

## DECLARATIONS

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### Data availability

The datasets used and/or analysed during the current study available from the corresponding author on reasonable request.

### Author's contribution

Z. Nazer: Conceptualization, methodology, computational analysis, data curation, formal analysis, validation, writing – original draft, review, and editing. P. Rabb: Conceptualization, methodology, art historical validation, supervision, project administration, resources, writing – review and editing, and validation.

### Competing interests

The authors declare no competing interests in this research and publication.

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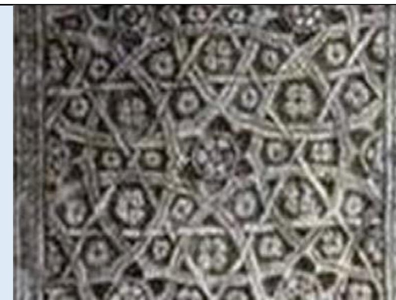
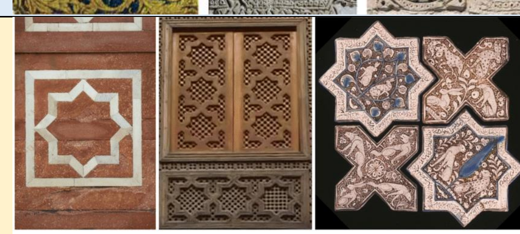
## Appendix

Table A1 presents the full list of 41 Islamic Geometric Patterns (IGPs) analyzed in this study, providing details on their dynastic origin, monument source, pattern type, symmetry group classification (using Orbifold notation), and computationally derived Symmetry Complexity Index (SCI) score. This appendix serves as a comprehensive data repository for reproducibility and further analysis.

**Table A1.** Full Pattern List of Analyzed Islamic Geometric Patterns (IGPs), (source: Authors)

| Dynasty | Pattern ID | Monument | Pattern Type | Symmetry Group (Orbifold) | SCI Score | Photo |
|---|---|---|---|---|---|---|
| Umayyad Dynasty (4 Patterns) | IGP_001 | Dome of the Rock, Qubbat As-Sakhrah | Vegetal Scroll | p1 | 0.0 |  |
| | IGP_002 | Dome of the Rock, Interior Mosaic | Vegetal & Geometric | p2 | 15.0 |  |
| | IGP_003 | Dome of the Rock, Exterior Tiles | Simple Rosette | p2 | 20.0 |  |
| | IGP_004 | Great Mosque of Damascus, Mosaic | Vegetal & Geometric | p2 | 18.5 |  |

| | | | | | | |
|---|---|---|---|---|---|---|
| **Abbasid Dynasty (6 Patterns)** | IGP_005 | Ibn Tulun Mosque, Window Grille | 6-Point Star | p6 | 45.0 |  |
| | IGP_006 | Ibn Tulun Mosque, Mihrab Decoration | 8-Point Star | p4 | 48.0 |  |
| | IGP_007 | Abbasid Palace, Baghdad | 6-Point Rosette | p6 | 47.5 |  |
| | IGP_008 | Great Mosque of Kairouan, Interior Panel | Simple Geometric Shapes | p2 | 30.0 |  |
| | IGP_009 | Samarra Stucco Panel | Abstract Geometric | p4 | 42.0 |  |
| | IGP_010 | Great Mosque of Cordoba, Mihrab | 8-Point Star Variation | p4 | 46.0 |  |
| **Fatimid Dynasty (5 Patterns)** | IGP_011 | Al-Azhar Mosque, Window Screen | 8-Point Star (rotated p4g) | p4g | 55.0 |  |
| | IGP_012 | Al-Azhar Mosque, Stucco Panel | 6-Point Star | p6 | 50.0 |  |

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|---|---|---|---|---|---|---|
| | IGP_013 | Al-Aqmar Mosque, Facade Ornament | 6-8 Point Star | p6 | 52.0 |  |
| | IGP_014 | Al-Salih Tala'i Mosque, Wall Decoration | 12-Point Star | p6 | 51.5 |  |
| **Seljuk Dynasty (8 Patterns)** | IGP_015 | Gunbad-i Qabus, Radial Flange | 4-Point (p4) | p4 | 33.3 |  |
| | IGP_016 | Kharāqan East Tower, Brick Ribs | 8-Point (p4m) | p4m | 70.0 |  |
| | IGP_017 | Kharāqan East Tower, Swastika Panel | 8-Point (p4g) | p4g | 65.0 |  |
| | IGP_018 | Kharāqan East Tower, Hexagonal Stars | 6-Point Star | p6 | 60.0 |  |

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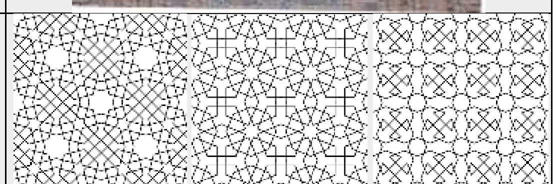
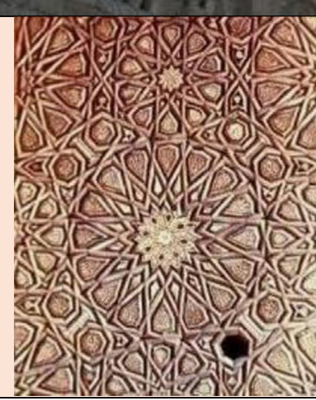
|---|---|---|---|---|---|---|
| | IGP_019 | Isfahan Jameh Mosque, South Dome | 10-13 Point Star | p10 | 75.0 |  |
| | IGP_020 | Barsian Friday Mosque, Minaret | 9-13 Point Star | p31m | 68.0 |   |
| | IGP_021 | Gunbad-i Surkh, Main Facade | 6-Point Star | p6 | 72.0 |  |
| | IGP_022 | Mausoleum of Yusif ibn Kuseyir, Dodecagons | 12-Point (p4m) | p4m | 74.0 |  |
| **Mamluk Dynasty (8 Patterns)** | IGP_023 | Qalawun Complex, Window Grille | 6-Point Star | p6 | 78.0 |  |
| | IGP_024 | Sultan Hassan Mosque, Minbar Panel | 16-Point Pattern | p4m | 85.0 |  |

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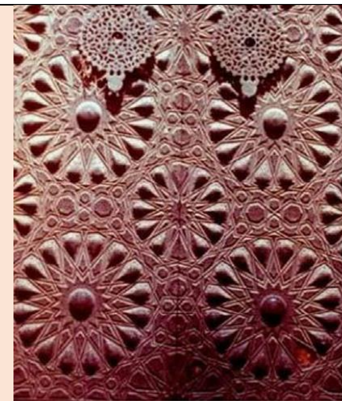
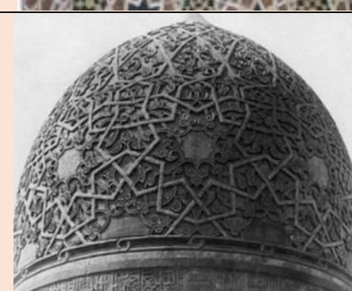
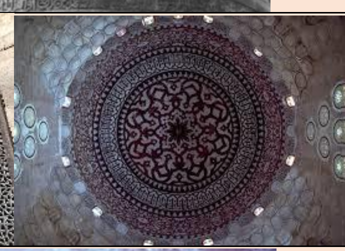
| | IGP_025 | Sultan Hassan Mosque, Entrance Door | 10-Point Star | p10 | 72.0 |  |
| | IGP_026 | Al-Nasir Muhammad Mosque, Mihrab Hood | 10-Point Star | p10 | 73.0 |  |
| | IGP_027 | Qaytbay Mosque, Dome Decoration | 16-Point Pattern | p4m | 83.0 |  |
| | IGP_028 | Khanqah of Faraj ibn Barquq, Wall Panel | 8-Point Star | p6m | 76.0 |   |
| | IGP_029 | Mosque of Baybars, Window Grille | 8-Point Star | p4g | 71.0 |  |

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| **Ottoman Dynasty (5 Patterns)** | IGP_030 | Yesil Mosque, Bursa, Wall Tiles | 6-Point Star | p6 | 58.0 |  |
| | IGP_031 | Yesil Mosque, Bursa, Ceiling Decoration | 8-Point Star | p4 | 60.0 |  |
| | IGP_032 | Bayezid II Complex, Portal Ornament | 10-Point Star | p10 | 62.0 |  |
| | IGP_033 | Selimiye Complex, Window Crown | 6-Point Star | p6 | 59.0 |  |

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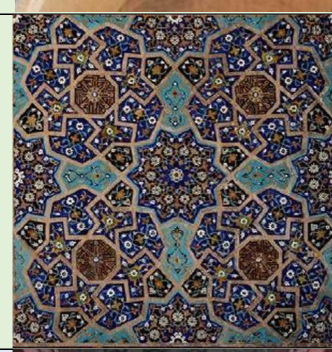
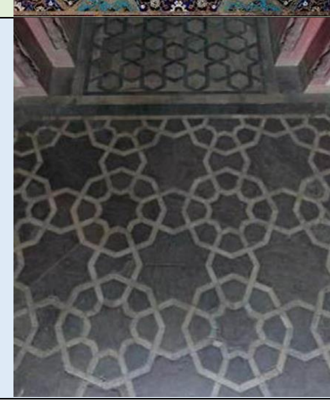
| Safavid Dynasty (4 Patterns) | IGP_034 | Ali Qapu Palace, Balcony Railing | 8-Point Star | p4g | 63.0 |  |
| | IGP_035 | Chehel Sutun Palace, Ceiling | 10-Point Star | p10 | 64.0 |  |
| | IGP_036 | Hakim Mosque, Isfahan, Facade Tilework | 8-Point Star | p4g | 61.5 |  |
| | IGP_037 | Friday Mosque of Isfahan, Mihrab | 10-Point Star | p10 | 63.5 |  |
| Mughal Dynasty (5 Patterns) | IGP_038 | Humayun's Tomb, Window Grille | 8-Point Star | p4g | 64.0 |  |

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| | IGP_039 | Red Fort Agra, Marble Inlay | 10-Point Star | p10 | 65.0 | |  |
| | IGP_040 | Fatehpur Sikri Mosque, Dome Pier | 14-Point Pattern | p10 | 67.0 | |  |
| | IGP_041 | Lahore Fort, Sheesh Mahal Ceiling | 12-Point Star | p6m | 66.0 | |  |