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Buckling Analysis of Biaxially Compressed All-Round Simply Supported (SSSS) Thin Rectangular Isotropicplates using the Galerkin's Method

David Ogbonna Onwuka[1], Owus Mathias Ibearugbulem[1], Stanley Emeka Iwuoha[1]✉, Joan Ijeoma Arimanwa[1] and Samuel Sule[2]

[1]*Department of Civil Engineering, Federal University of Technology, Owerri, Imo State, Nigeria*

[2]*Department of Civil and Environmental Engineering, University of Port Harcourt, Rivers State, Nigeria*

Corresponding author's Email: iwuohastanley1@gmail.com, samvictoryahead@yahoo.com

ABSTRACT: This work studied the buckling analysis of biaxially compressed all-round simply supported (SSSS) thin rectangular isotropic plates using the Galerkin's method. The study was limited to thin rectangular isotropic plates having aspect ratios ranging from 1 to 2. The general equation for the critical buckling load of a biaxially loaded plate, was formulated from the overall governing differential equation for plates, using the Galerkin's method. The derived general equation, was expressed as the load in the x-axis in terms of that in the y-axis. This was done by means of a linear relationship which was obtained for the buckling load on the y- axis in terms of that on the x-axis. The SSSS plate deflection equation, was obtained using the polynomial series, and was substituted into the general equation of the critical buckling load of a biaxially loaded plate. This yielded the unique expression for the critical buckling load of a biaxially loaded SSSS plate. Different values (0 to 2) of aspect ratios and "k" (relationship constant between forces on the Y-axis and forces on the X-axis) values (0.1 to 1) were substituted into the critical buckling equation for an SSSS plate, and the critical buckling load coefficients were obtained. The critical buckling load coefficient of a square plate (i.e. at k equal to 1), was obtained as 19.754. When compared with the exact value (19.744) obtained by other researchers who used the trigonometric series, a percentage difference of 0.047 was discovered. At k equal to zero, and for different aspect ratios, the results of the present study showed a maximum percentage difference of 0.069 with those given in the literature. It was therefore concluded that this paper has provided the critical buckling load equation, and results for a biaxially loaded SSSS plates having different k values and aspect ratios, whose result has not been found in the literature.

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Abbreviations: a:Length of the plate; b:Width of the plate; A:Coefficient of deflection of the plate; w:Deflection equation of the plate; w''^{R}, w''^{Q}: Second derivatives of the deflection equation in the R and Q directions respectively; H: Plate Shape function; D: Flexural Rigidity of the plate;$\propto$: Aspect Ratio (b/a); $a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3, b_4$:Polynomial constants; h: Thickness of the plate; N_x: Load applied in the x-direction; N_y: load applied in the y-direction; N_{xcr}: Critical buckling load in the x-direction; X:Primary axis of the plate; Y: secondary axis of the plate; S: Simple support; C: Clamped support; R: non- dimensional parameter on the x-axis equal to x/a; Q: Non-dimensional parameter on the y-axis equal to y/b ; k: constant relating N_y and N_x; Nxi: critical buckling load coefficient at k = 0.i.

INTRODUCTION

Plates are widely used Engineering materials. They find relevance in Civil, Structural, Mechanical and Aeronautic Engineering. Due to the importance and wide application of this structural material, several researches have been carried out with the aim of maximizing its potentials for wider structural applications. One area of such research is the buckling analysis of plates. When a thin rectangular plate is subjected to in-plane compressive loads, and the loads are gradually increased, the plate gradually loses its stability and begins to buckle (pass from the state of stable equilibrium to a state of unstable equilibrium) at a critical value of the compressive loads even when transverse loads are not applied. The determination of the critical buckling loads is essential so as to ensure that the use of the plate for any Engineering purpose is safe.

Several works on the buckling analysis of plates had been done in the past. Chajes (1974) showed that the buckling load of a plate which is simply supported all round and uniformly compressed in one direction is given by;

$$N_x = \frac{D\pi^2}{b^2}\left(\frac{mb}{a} + \frac{n^2 a}{mb}\right)^2 \quad (1)$$

And for a plate fully clamped on all sides and uniaxially loaded, the buckling load is given by

$$N_x = \frac{10.67\,\pi^2 D}{a^2} \quad (2)$$

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where N_x is the buckling load on the x-axis; "a" and "b" are the plate dimensions in the x and y axes respectively; "m" and "n" are the number of half-waves that the plate buckles into; and D is the flexural rigidity of the plate.

Ibearugbulem et al. (2014) used a polynomial shape function in the Ritz method for buckling analysis of SSSS plates under uniaxial compressive loads. Jayashankarbabu et al. (2013) used the finite element method to obtain the elastic buckling load factors for square plates with different boundary conditions (i.e. SCSC, CCCC, SSSS) containing square and circular cutouts and subjected to uniaxial compression, with the load applied at the simply supported and at the clamped edges. Li-keet al. (2015) proposed a new method which does not require the global stiffness matrix of the system, but reduces the system matrix order and improves the computational efficiency for analyzing plates, which are simply supported on all edges. Ezeh et al. (2014) proposed shape functions based on the characteristic orthogonal polynomial and used them in the Galerkin's indirect variational principle for elastic buckling analysis of a thin plate clamped at all edges and subjected to axial load in the x-direction. Ventsel and Krauthermmer (2001), Iyengar (1988) and Chajes (1974), individually demonstrated that for a biaxially loaded square SSSS plate subjected to uniform stress on both sides, the critical buckling load Ncr, is given by;

$$N_{cr} = \frac{2\,\pi^2 D}{a^2} \quad (3)$$

Where: D is the flexural rigidity of the plate and "a" is the plate length.

From literature, it is clear that research works on the buckling analysis of plates had revolved around analysis of uniaxially loaded plates and biaxially loaded square plates subject to uniform stress using trigonometric functions. To the best of our knowledge, works have not been done on the buckling analysis of SSSS plates subject to biaxial loading for rectangular plates using polynomial shape functions. Besides, plates having aspect ratios different from 1 and subject to different values of the loads in the both axes (load on the x-axis being different from that on the y-axis) have not been considered previously. The objective of this work is to fill the gap in literature by providing solutions to the buckling analysis of SSSS rectangular plates subject to biaxial loadings (such that forces in the x-axis are not the same as those in the y-axis) using the polynomial shape function proposed by Ibearugbulem (2012) in the Galerkin's method.

MATERIALS AND METHODS

The method of solution applied in this work is purely mathematical, and involves the following stages as presented below.

Formulation of the Equation for the Buckling of a Biaxially Compressed Thin Rectangular Isotropic Plate:

Consider a simply supported flat isotropic plate undergoing buckling under the action of biaxial in-plane loads as shown in Figure 1. Let us assume that the thickness of the plate in the z- direction, is far smaller than the length and width of the plate in the x- and y-directions respectively.

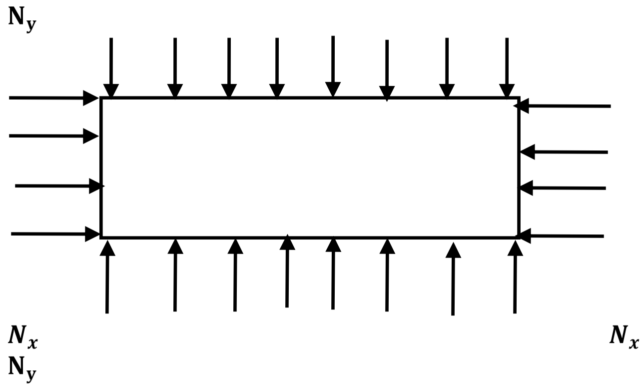

Figure 1. Simply supported isotropic flat plate under biaxial in-plane loads

As given by Ibearugbulem et al. (2014), the overall governing differential equation for plates is given as Equation 4.

$$q - N_x\left(\frac{\partial^2 w}{\partial x^2}\right) - 2N_{xy}\left(\frac{\partial^2 w}{\partial x \partial y}\right) - N_y\left(\frac{\partial^2 w}{\partial y^2}\right) + m\lambda^2 w = D\left[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right] \quad (4)$$

Where:

q is the lateral load acting on the plates and causing the plates to bend,

N_{xy} is the shear force acting on the plate,

N_x *and* N_y, are theaxial forces applied in the x and y- directions respectively,

$m\lambda^2 w$ is the force due to vibration of the plate,

w is the deflection of the plate defined by Equation 5.

$$w = AH \quad (5)$$

For plates undergoing biaxial buckling, the lateral load (q), the shear force (Nxy) , and the force due to vibration $(m\lambda^2 w)$, will all be zero. Hence, the critical buckling equation for plates undergoing biaxial buckling is as given by Equation 6.

$$-N_x\left(\frac{\partial^2 w}{\partial x^2}\right) - N_y\left(\frac{\partial^2 w}{\partial y^2}\right) = D\left[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right] \quad (6)$$

Equation (6), is an equation of the biaxial forces acting on the plate. These forces (both internal and

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external), act to deform the plate. If the average deformation on the plate by the forces is "w", then the work done by the forces while acting on the plate, shall be obtained by multiplying Equation (6) by "w". This yields the work equation as Equation 7.

$$-N_x\left(\frac{\partial^2 w}{\partial x^2}\right)w - N_y\left(\frac{\partial^2 w}{\partial y^2}\right)w = D\left[\frac{\partial^4 w}{\partial x^4}.w + 2w.\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}.w\right] \quad (7)$$

Equation (7) is the Galerkin's expression for the buckling analysis of plates subjected to biaxial forces.

Integrating Equation (7) along the x and y axes respectively, yields the entire work equation of the plate, as Equation 8.

$$-N_x \iint \left(\frac{\partial^2 w}{\partial x^2}\right).w\partial x\partial y - N_y \iint \left(\frac{\partial^2 w}{\partial y^2}\right).w\,\partial x\partial y = D\iint \left[\frac{\partial^4 w}{\partial x^4}.w + 2w\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}.w\right]\partial x\partial y \quad (8)$$

For easy solution of the plate problem, let us consider the diagram in Figure 2.

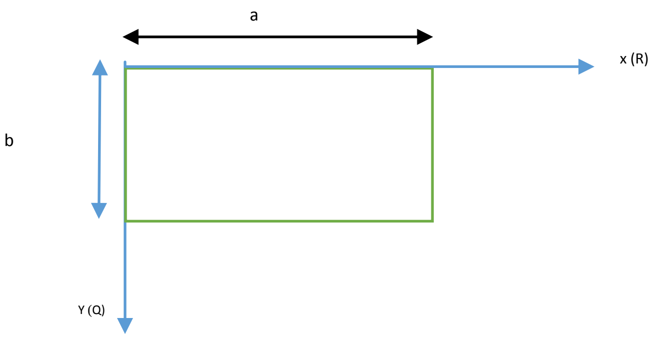

Figure 2: Plate Diagram showing plate nomenclature

The Cartesian co-ordinates are expressed in terms of non-dimensional parameters, R and Q, as given by Equation 9.

$$R = \frac{x}{a}, \qquad Q = \frac{y}{b} \qquad (9)$$

Let the aspect ratio $\propto$, of the plate, be given by Equation (10).

$$\propto = \frac{b}{a} \qquad (10)$$

Substituting Equations (5), (9) and (10) into Equation (8) and simplifying it, yields Equation (11).

$$-\frac{N_x}{a^2}\iint\left(\frac{\partial H}{\partial R}\right)^2 \partial R\partial Q - \frac{N_y}{a^2 \propto^2}\iint\left(\frac{\partial H}{\partial Q}\right)^2 \partial R\partial Q = \frac{D}{a^4}\iint\left[\left(\frac{\partial^4 H}{\partial R^4}\right)H + \frac{2H}{\propto^2}\left(\frac{\partial^4 H}{\partial R^2 \partial Q^2}\right) + \frac{H}{\propto^4}\left(\frac{\partial^4 H}{\partial Q^4}\right)\right]\partial R\partial Q \quad (11)$$

Let Equation 12, be the equation which relates the force in the x and y axes of the plate.

$$N_y = KN_x \qquad (12)$$

Hence, substituting Equation (12) into Equation (11), gives Equation (13).

$$-\frac{N_x}{a^2}\iint\left(\frac{\partial H}{\partial R}\right)^2 \partial R\partial Q - \frac{KN_x}{a^2 \propto^2}\iint\left(\frac{\partial H}{\partial Q}\right)^2 \partial R\partial Q = \frac{D}{a^4}\iint\left[\left(\frac{\partial^4 H}{\partial R^4}\right)H + \frac{2H}{\propto^2}\left(\frac{\partial^4 H}{\partial R^2 \partial Q^2}\right) + \frac{H}{\propto^4}\left(\frac{\partial^4 H}{dQ^4}\right)\right]\partial R\partial Q \quad (13)$$

Multiplying Equation (13) by a^2, and making N_x the subject of the formula, gives the general equation used for determining the critical buckling loads, N_x, of a biaxially compressed thin rectangular isotropic plate as Equation (14).

$$N_x = -\frac{D/a^2 \iint\left[\left(\frac{\partial^4 H}{\partial R^4}\right)H + \frac{2H}{\propto^2}\left(\frac{\partial^4 H}{\partial R^2 \partial Q^2}\right) + \frac{H}{\propto^4}\left(\frac{\partial^4 H}{\partial Q^4}\right)\right]\partial R\partial Q}{\iint\left[\left(\frac{\partial H}{\partial R}\right)^2 + \frac{K}{\propto^2}\left(\frac{\partial H}{\partial Q}\right)^2\right]\partial R\partial Q} \quad (14)$$

Derivation of the Deflection Equation of an all-round simply supported isotropic plate:

For an all-round simply supported plate, the deflection and moments are zero at all points along the supports of the plate. Thus the boundary conditions of the SSSS plate are as follows;

$$w(R=0) = w''^R(R=0) = 0 \qquad (15)$$

$$w(R=1) = w''^R(R=1) = 0 \qquad (16)$$

$$w(Q=0) = w''^Q(Q=0) = 0 \qquad (17)$$

$$w(Q=1) = w''^Q(Q=1) = 0 \qquad (18)$$

Ibearugbulem (2011), assumed functions that satisfy the governing equation for plates. He expanded them on the Taylor-Mclaurin's series assuming the deflection equation "w", to be continuous and differentiable and obtained the general polynomial deflection equation for rectangular plates as Equation (19).

$$w = \sum_{m=0}^{4}\sum_{n=0}^{4} a_m b_n R^m.Q^n \qquad (19)$$

In the expanded form, Equation (19) is given as Equation (20).

$$w = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4)(b_0 + b_1Q + b_2Q^2 + b_3Q^3 + b_4Q^4) \qquad (20)$$

Where:

$a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3, and\ b_4$ are constants; and R and Q are as already defined.

Substituting Equation (15) and Equation (16) into Equation (20), and solving the resulting simultaneous equations, yield the results given in Equation (21).

$$a_0 = a_2 = 0; a_3 = -2a_4\ and\ a_1 = a_4 \qquad (21)$$

Similarly, substituting Equation (17) and Equation (18) into Equation (20) gives Equation (22)

$$b_0 = b_2 = 0; b_3 = -2b_4\ and\ b_1 = b_4 \qquad (22)$$

Substituting Equations (21) and (22),into Equation (20), gives the particular deflection equation for anall-round simply supported thin rectangular isotropic plate as Equation (23)

$$w = a_4b_4(R - 2R^3 + R^4)(Q - 2Q^3 + Q^4) \qquad (23)$$

From Equation (5), w is given as, equal to A*H; where A, and H are as previously defined; and they are given by Equations 24 and 25 respectively.

$$A = a_4 b_4 \tag{24}$$

$$H = (R - 2R^3 + R^4)(Q - 2Q^3 + Q^4) \tag{25}$$

Application of the Formulated Equation to an All-Round Simply Supported Plate:

Differentiating Equation (25) with respect to R and Q, gave the results presented in Equations 26 to 28. Multiplying the derivatives of Equation (25) (with respect to R and Q) by the plate's shape function (H), gave the results in Equations 29 and 30.

$$\left(\frac{\partial H}{\partial R}\right)^2 = (1 - 12R^2 + 8R^3 + 36R^4 - 48R^5 + 16R^6)(Q^2 - 4Q^4 + 2Q^5 + 4Q^6 - 4Q^7 + Q^8) \tag{26}$$

$$\left(\frac{\partial H}{\partial Q}\right)^2 = (R^2 + 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8)(1 - 12Q^2 + 8Q^3 + 36Q^4 - 48Q^5 + 16Q^6) \tag{27}$$

$$\left(\frac{\partial^4 H}{\partial R^4}\right) = 24(R - 2R^3 + R^4)(Q^2 - 4Q^4 + 2Q^5 + 4Q^6 - 4Q^7 + Q^8) \tag{28}$$

$$\left(\frac{\partial^4 H}{\partial Q^4}\right) H = 24(R^2 - 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8)(Q - 2Q^3 + Q^3) \tag{29}$$

$$\left(\frac{\partial^4 H}{\partial R^2 \partial Q^2}\right) H = (-12R^2 + 12R^3 + 24R^4 - 36R^5 + 12R^6)(-12Q^2 + 12Q^3 + 24Q^4 - 36Q^5 + 12Q^6) \tag{30}$$

Integrating Equations (26) – (30) in a closed domain (from 0-1) with respect to R and Q, gave the numerical values expressed in Equations (31) to (35).

$$\int_0^1 \int_0^1 \left(\frac{\partial H}{\partial R}\right)^2 \partial R \partial Q = 0.0239. \tag{31}$$

$$\int_0^1 \int_0^1 \left(\frac{\partial H}{\partial Q}\right)^2 \partial R \partial Q = 0.0239. \tag{32}$$

$$\int_0^1 \int_0^1 \left(\frac{\partial^4 H}{\partial R^4}\right) H \partial R \partial Q = 0.2362. \tag{33}$$

$$\int_0^1 \int_0^1 \left(\frac{\partial^4 H}{\partial Q^4}\right) H \partial R \partial Q = 0.2362. \tag{34}$$

$$\int_0^1 \int_0^1 \left(\frac{\partial^4 H}{\partial R^4 \partial Q^2}\right) H \partial R \partial Q = 0.2359. \tag{35}$$

Substituting the numerical values obtained from Equations (31)-(35) into Equation (14), yields the critical buckling load equation for an all-round simply supported rectangular isotropic plate as Equation (36).

$$N_{xcr} = -\frac{D/_{a^2}\left[0.2362 + \frac{2}{\propto^2}(0.2359) + \frac{1}{\propto^4}(0.2362)\right]}{\left[0.0239 + \frac{k}{\propto^2}(0.0239)\right]} \tag{36}$$

But the general expression for the determination of the buckling coefficient of an all-round simply supported plate is given by Equation (37).

$$F = \frac{\left[0.2362 + \frac{2}{\propto^2}(0.2359) + \frac{1}{\propto^4}(0.2362)\right]}{\left[0.0239 + \frac{k}{\propto^2}(0.0239)\right]} \tag{37}$$

RESULTS AND DISCUSSION

Substituting different values of the plate aspect ratio and the constant "k" (which relates the buckling load on the y-axis to that on the x-axis), into Equation (37), yielded the results of the critical buckling load coefficients for a biaxially compressed all-round simply supported (SSSS) thin rectangular isotropic plate as presented in Table 1.

Where,

$N_x, N_{x1}, N_{x2}, N_{x3}, N_{x4}, N_{x5}, N_{x6}, N_{x7}, N_{x8}, N_{x9}$ *and* N_{x10} are the critical buckling loads at k equals 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0, respectively.

To validate the accuracy of the results obtained in this research work, the results obtained by Megson (2010), Ventsel and Krauthermmer (2001), and Ibearugbulem (2014), at k=0 (i.e. foruniaxially loaded plates), were compared with the present results in Table 2. And the results of the critical buckling load coefficients at $k = \propto = 1$ (i.e. for biaxially loaded plates under uniform stress on both axes), were also compared with those gotten by previous researchers (who used the trigonometric shape function) in Table 3.

It is seen from Table 2 that, the results of this present work agrees very closely with previous results of uniaxially loaded all-round simply supported plates subject to uniform stress along the x-axis (i.e. At k = 0), for different aspect ratios; as the maximum percentage difference between the present study and previous works is 0.057%. This percentage difference validates the results of the critical buckling load coefficients for other "k" values as presented on Table 1 for which there are no existing results to compare with, in literature.

Table 1: Critical Buckling Load Coefficients for biaxially loaded SSSS Plates

Aspect ratio	Nx	Nx1	Nx2	Nx3	Nx4	Nx5	Nx6	Nx7	Nx8	Nx9	Nx10
1	39.5075	35.9159	32.9229	30.3904	28.2197	26.3384	24.6922	23.2397	21.9486	20.7934	19.7538
1.1	32.9485	30.4334	28.275	26.4025	24.7626	23.3145	22.0264	20.8732	19.8347	18.8947	18.0397
1.2	28.3585	26.517	24.9001	23.4691	22.1936	21.0496	20.0178	19.0823	18.2305	17.4514	16.7362
1.3	25.0247	23.6266	22.3766	21.2521	20.2353	19.3113	18.468	17.6953	16.9846	16.3288	15.7218
1.4	22.5278	21.4342	20.4419	19.5374	18.7095	17.949	17.2478	16.5994	15.998	15.4386	14.9171
1.5	20.6092	19.7322	18.9268	18.1846	17.4983	16.862	16.2704	15.7189	15.2035	14.7208	14.2679
1.6	19.1025	18.3844	17.7183	17.0987	16.5211	15.9812	15.4754	15.0007	14.5543	14.1336	13.7366
1.7	17.8972	17.2986	16.7388	16.2141	15.7213	15.2575	14.8203	14.4075	14.0171	13.6472	13.2964
1.8	16.9174	16.4109	15.9339	15.4838	15.0584	14.6558	14.2741	13.9118	13.5675	13.2397	12.9275
1.9	16.1098	15.6756	15.2642	14.8738	14.5029	14.15	13.8139	13.4934	13.1874	12.895	12.6153
2	15.436	15.0595	14.7009	14.3591	14.0327	13.7209	13.4226	13.137	12.8633	12.6008	12.3488

Table 2: Critical Buckling load coefficients for an SSSS rectangular plate under uniform Unilateral stress (i.e. at k = 0)

Aspect Ratios $(\propto = \frac{b}{a})$	Ventsel and Krauthermmer (2001), Megson (2010), and Chajes (1974), Results N_1	Ibearugbulem et al (2014), N_2	Present Results N_3	Percentage Difference $\left(\frac{N_3 - N_2}{N_3}\right)x100$	Percentage Difference $\left(\frac{N_3 - N_1}{N_3}\right)x100$
1	39.488	39.507	39.5075	0.00126	0.047
1.1	32.932	32.948	32.9485	0.00151	0.048
1.2	28.344	28.358	28.3584	0.00141	0.049
1.3	25.011	25.025	25.0246	0.00159	0.051
1.4	22.515	22.528	22.5278	-0.0089	0.054
1.5	20.597	20.609	20.6091	0.00048	0.057
1.6	19.091	19.103	19.1025	-0.0026	0.059
1.7	17.886	17.898	17.8972	-0.0045	0.062
1.8	16.906	16.917	16.9174	0.00236	0.064
1.9	16.099	16.110	16.109	-0.0062	0.067
2	15.425	15.436	15.43598	-0.00013	0.069

Table 3: Critical Buckling load coefficients for square SSSS plates under uniform biaxial stress (i.e. at $\propto =$ k=1)

Author	Critical Buckling Load for a Biaxially Loaded SSSS square Plates under uniform stress	Percentage difference
Ventsel and Krauthermmer (2001), Chajes (1974), Iyengar (1988)	$Ncr = \frac{2\,\pi^2 D}{a^2} = \frac{19.744D}{a^2}$	0.047%
Present study	$Ncr = \frac{2.000959\,\pi^2 D}{a^2} = \frac{19.753D}{a^2}$	

It can be seen from the insignificant percentage difference between the results obtained in this research work and those obtained by Ventsel and Krauthermmer (2001), Chajes (1974) and Iyengar (1988) as shown in Table 3, that, the polynomial shape functions used for the SSSS plate, under biaxial loading, accurately defines the plate's deformed shape (given that the results obtained in the present study agrees very closely with the results obtained by previous researchers who used the trigonometric shape functions). Since the results obtained from this work agrees with the results of the others mentioned for k=1, then the results obtained in this work for other k values (for which there are no other existing results to compare with in literature) must also be accurate.

CONCLUSION

From the study, the following conclusions can be drawn.

i. This paper has provided the critical buckling load results for a biaxially loaded all-round simply supported thin rectangular isotropic plates with different aspect ratios and k values.

ii. Since the results obtained from this work agrees with the results of the other researchers (such as Ventsel and Krauthermmer, 2001; Chajes, 1974; Iyengar, 1988), for k=0, and k=1 respectively, then the results obtained in this work for other k values (for which there are no other existing results to compare with, in literature), must also be accurate.

iii. Since the accuracy of the buckling load, is dependent on the accuracy of the assumed shape function used, it can be clearly said that the polynomial shape functions used in this work for the SSSS plate, accurately defines the plate's deformed shape under biaxial loading.

iv. This also shows that the presently formulated equation for the buckling analysis of biaxially compressed all-round simply supported (SSSS) thin rectangular isotropic plates, is accurate

v. The critical buckling loads for different aspect ratios decreased with increase in the k values and aspect ratios respectively.

Recommendation

i. The equation for the determination of the critical buckling load of an all-round simply supported biaxially loaded thin rectangular isotropic plate developed in this work, is recommended for quick and easy analysis of the critical buckling load of simply supported plates with aspect ratios not covered in this work.

ii. The critical buckling loads coefficients presented in Table 1 of this work, is recommended for use, as a quick reference guide for the determination of the critical buckling loads for an all-round simply supported thin rectangular isotropicplate (which is biaxially loaded) in the design of plated structures.

Competing interests

The authors declare that they have no competing interests.

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