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Prediction of Seasonal and Annual Rainfall using Order Statistics Approach of Gumbel and Frechet Distributions

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ABSTRACT: Prediction of seasonal and annual rainfall for a river basin has utmost importance for planning of irrigation and drainage systems as also for command area development. This can be achieved by deterministic, conceptual and probabilistic methods. This paper illustrates the use of Gumbel and Frechet probability distributions for modelling seasonal and annual rainfall for Krishna and Godavari river basins. Lieblein technique based on order statistics approach is used for determination of parameters of the distributions. Model performance indicators such as correlation coefficient, model efficiency and root mean square error are used for the selection of suitable distribution for modelling seasonal and annual rainfall. The study suggests the Gumbel distribution is better suited for prediction of seasonal and annual rainfall for Krishna and Godavari river basins.

Keywords: Correlation, Frechet, Gumbel, Mean square error, Model efficiency, Rainfall

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INTRODUCTION

Prediction of seasonal and annual rainfall for a river basin has utmost importance for planning of irrigation and drainage systems as also for command area development. Since the distribution of rainfall varies over space and time, it is required to analyze the data covering long periods and recorded at various locations to arrive at reliable information for decision support (IAEA, 2003). Further, such data need to be analyzed in different ways, depending on the issue under consideration. For example, analysis of consecutive days of rainfall is more relevant for drainage design of agricultural lands, whereas analysis of weekly rainfall data is relevant for planning of cropping pattern. Likewise, analysis of monthly, seasonal and annual data is more useful for water management practices (May, 2004). In practice, deterministic, conceptual and probabilistic methods are generally used for prediction of rainfall (Gumbel, 1960). In this paper, probabilistic method is used.

In probabilistic theory, generalized extreme value distribution is identified as a family of continuous probability distributions that include Gumbel, Frechet and Weibull (Arora and Singh, 1987). Research studies detailed that the extreme value distributions are widely applied for assessment of meteorological variables such as rainfall, evaporation, temperature and wind speed, etc; and therefore adopted in the present study (Manik and Datta, 1998; Singh et al., 2001; Lee and Heo, 2011; Xu et al., 2012). Standard procedures like Method of Moments (MoM), Probability Weighted Moments (PWM), Maximum Likelihood Method (MLM), Method of Least Squares (MLS) and Lieblein technique based on Order Statistics Approach (OSA) are commonly available for determination of parameters of extreme value distributions. Lieblein (1974) suggested the OSA for determination of parameters of Gumbel and Frechet distributions for modelling meteorological data. He also described that the parameters determined by OSA are unbiased and having minimum variance. Landwehr et al. (1979) discussed that MLM may produce quality estimators in small samples, especially when the random variable is restricted to an interval that depends on the parameters. Raynal and Salas (1986) analyzed that MoM estimates are usually inferior in quality because higher order moments are more likely to be highly biased in relative small samples. Phien (1987) studied that MLM is considered the most efficient method, since it provides the smallest sampling variance of the estimated parameters and hence of the estimated quantiles compared to other methods. He also studied that the method has the disadvantage of frequently giving biased estimates and often failed to give the desired accuracy in estimating extremes from hydrological data. Rasmussen and Gautam (2003) described that the parameter estimates from small samples using MLS and PWM are sometimes more accurate than the MLM estimates and less than the OSA estimates for Gumbel distribution. Since there is no general agreement in applying a particular method for determination of parameters of extreme value distributions, an attempt is made for prediction of seasonal and annual rainfall for Krishna and Godavari

basins using OSA because of the characteristics of the parameters. In the present study, Weibull is not considered for prediction of rainfall because of non-existence of OSA for determination of parameters of the distribution. Model Performance Indicators (MPIs) such as Correlation Coefficient (CC), Model Efficiency (MEF) and Root Mean Square Error (RMSE) are used for the selection of suitable distribution for modelling rainfall data (Sharda and Das, 2005; Vaidya et al., 2008; Sreekala et al., 2012). The methodology adopted in prediction of rainfall using Gumbel and Frechet distributions, and computation of MPIs is briefly described in the ensuing sections.

MATERIAL AND METHODS

Probability Distributions

The Cumulative Distribution Function (CDF) of Gumbel and Frechet distributions are given by:

$F(X) = e^{-e^{-\left(\frac{X_i-\alpha_G}{\beta_G}\right)}}$, $\beta_G>0$ and $-\infty<X_i<\infty$ (Gumbel)

$F(X) = e^{-\left(\frac{X_i}{\beta_F}\right)^{(-\lambda_F)}}$, $\beta_F>0$ and $-\infty<X_i<\infty$ (Frechet)

where, α_G and β_G are the location and scale parameters of Gumbel distribution. The rainfall estimates (X_G) at different probability levels (P) adopting Gumbel distribution are computed from $X_G = \alpha_G + Y_T\beta_G$ with Y_T=-ln(-ln(1-P)). Similarly, β_F and λ_F are the scale and shape parameters of Frechet distribution. Based on extreme value theory, Frechet distribution can be transformed to Gumbel distribution through logarithmic transformation. Under this transformation, the rainfall estimates (X_F) adopting Frechet distribution are computed from $X_F = Exp(X_G)$, $\beta_F = Exp(\alpha_G)$ and $\lambda_F = 1/\beta_G$ (Suhaila and Jemain, 2007).

Order Statistics Approach

The approach is based on the assumption that the set of extreme values constitutes a statistically independent series of observations. The parameters of Gumbel distribution are given by:

$\alpha_G = r^*\alpha_M^* + r'\alpha_M'$; $\beta_G = r^*\beta_M^* + r'\beta_M'$

where r^* and r' are proportionality factors, which can be obtained from the selected values of k, n and n′ using the relations $r^* = kn/N$ and $r' = n'/N$. Here, N is the sample size contains basic data that are divided into k sub groups of n elements each leaving n′ remainders.

In OSA, α_M^* and β_M^* are the distribution parameters of the groups and α_M' and β_M' are the parameters of the remainders, if any. These can be computed from the following equations:

$\alpha_M^* = (1/k)\sum_{i=1}^{n}\alpha_{ni}S_i$ and $\alpha_M' = \sum_{i=1}^{n'}\alpha_{n'i}X_i$

$\beta_M^* = (1/k)\sum_{i=1}^{n}\beta_{ni}S_i$ and $\beta_M' = \sum_{i=1}^{n'}\beta_{n'i}X_i$

where $S_i = \sum_{i=1}^{k}X_{ij,}$ j=1,2,3,..,n. The weights of α_{ni} and β_{ni} are given in Table 1(AERB, 2008).

Table 1. Weights of α_{ni} and β_{ni} used in OSA for determination of parameters of Gumbel and Frechet distributions

α_{ni} (or) β_{ni}	i					
	1	2	3	4	5	6
α_{2i}	0.91637	0.08363				
α_{3i}	0.65632	0.25571	0.08797			
α_{4i}	0.51099	0.26394	0.15368	0.07138		
α_{5i}	0.41893	0.24628	0.16761	0.10882	0.05835	
α_{6i}	0.35545	0.22549	0.16562	0.12105	0.08352	0.04887
β_{2i}	-0.72135	0.72135				
β_{3i}	-0.63054	0.25582	0.37473			
β_{4i}	-0.55862	0.08590	0.22392	0.24879		
β_{5i}	-0.50313	0.00653	0.13046	0.18166	0.18448	
β_{6i}	-0.45927	-0.03599	0.07319	0.12672	0.14953	0.14581

Model Performance Indicators

The performance of predicted seasonal and annual rainfall using Gumbel and Frechet distributions are analyzed by CC, MEF and RMSE, which are described as:

$$CC = \frac{\sum_{i=1}^{N}\left(X_i - \overline{X}\right)\left(X_i^* - \overline{X^*}\right)}{\sqrt{\left(\sum_{i=1}^{N}\left(X_i - \overline{X}\right)^2\right)\left(\sum_{i=1}^{N}\left(X_i^* - \overline{X^*}\right)^2\right)}} \quad \ldots (1)$$

$$MEF(\%) = \left(1 - \frac{\sum_{i=1}^{N}\left(X_i - X_i^*\right)^2}{\sum_{i=1}^{N}\left(X_i - \overline{X}\right)^2}\right)*100 \quad \ldots (2)$$

$$RMSE = \left((1/N)\sum_{i=1}^{N}\left(X_i - X_i^*\right)^2\right)^{0.5} \quad \ldots (3)$$

where X_i is the recorded rainfall of i^{th} event, X_i^* is the predicated rainfall of i^{th} event, $\overline{X}$ is the average value of recorded rainfall and $\overline{X^*}$ is the average value of predicted rainfall (Chen and Adams, 2006).

Application

In this paper, an attempt is made to predict the seasonal and annual rainfall for Krishna and Godavari river basins using Gumbel and Frechet distributions. The drainage area of Krishna and Godavari basins are 295650 km^2 and 330628 km^2. The average annual rainfall for Krishna and Godavari basins are 825.7 mm and 1068.3 mm respectively. Daily rainfall data recorded at the river basins for the period 1901 to 2005 is used (IITM, 2007). From the analysis of historical rainfall data, it is observed that the percentages of rainfall received during monsoon (June to September), post-monsoon (October to December), winter (January and February) and summer (March to May) seasons, with reference to annual rainfall are 70.4%, 18.9%, 1.2% and 9.5% respectively for Krishna basin. Similarly, the percentages of rainfall received during monsoon, post-monsoon, winter and summer seasons are noted to be 84.4%, 8.8%, 2.1% and 4.7% respectively for Godavari basin.

RESULTS AND DISCUSSIONS

By using the procedures described above, a computer program was developed and used to fit the recorded data for prediction of seasonal and annual rainfall using Gumbel and Frechet distributions. The program computes the parameters of the probability distributions using OSA, predicated seasonal and annual rainfall at different probability levels and values of MPIs. Tables 2 and 3 give the descriptive statistics for the series of predicted seasonal and annual rainfall using Gumbel and Frechet for Krishna and Godavari basins respectively. The time series plots of recorded and predicted seasonal and annual rainfall using Gumbel and Frechet distributions for Krishna and Godavari river basins are presented in Figures 1 to 3.

From Tables 2 and 3, it may be noted that the percentage of variations on the average predicted seasonal and annual rainfall using Gumbel and Frechet, with reference to the average recorded rainfall, are noted to be about 2% to 3% for Krishna basin and about 5% to 6% for Godavari.

Performance Analysis

To evaluate the performance of the predicted seasonal and annual rainfall using Gumbel and Frechet distributions for Krishna and Godavari river basins, values of MPIs were computed from Eqs. (1-3), and given in Tables 4 and 5.

Table 2. Descriptive statistics of recorded and predicted rainfall using Gumbel and Frechet distributions for Krishna basin

Rainfall series	Descriptive statistics					
	Average (mm)			Standard deviation (mm)		
	Recorded rainfall	Predicted rainfall		Recorded rainfall	Predicted rainfall	
		Gumbel	Frechet		Gumbel	Frechet
Monsoon	622.9	605.3	607.4	132.3	121.1	141.7
Post-monsoon	167.2	162.5	163.0	35.5	32.5	38.0
Winter	10.6	10.3	10.4	2.3	2.1	2.4
Summer	84.1	81.7	82.0	17.8	16.3	19.1
Annual	884.8	859.8	862.8	187.9	172.0	201.2

Table 3. Descriptive statistics of recorded and predicted rainfall using Gumbel and Frechet distributions for Godavari basin

Rainfall series	Descriptive statistics					
	Average (mm)			Standard deviation (mm)		
	Recorded rainfall	Predicted rainfall		Recorded rainfall	Predicted rainfall	
		Gumbel	Frechet		Gumbel	Frechet
Monsoon	859.0	811.9	812.3	175.5	147.7	166.1
Post-monsoon	89.6	84.6	84.7	18.3	15.4	17.3
Winter	21.4	20.3	20.2	4.4	3.9	4.1
Summer	47.8	45.2	45.2	9.8	8.2	9.2
Annual	1017.8	962.0	962.4	208.0	175.2	196.7

Table 4. Values of MPIs given by Gumbel and Frechet distributions for seasonal and annual rainfall patterns of Krishna river basin

Rainfall series	Values of MPIs					
	Gumbel			Frechet		
	CC	MEF (%)	RMSE (mm)	CC	MEF (%)	RMSE (mm)
Monsoon	0.991	95.868	26.5	0.961	90.526	41.0
Post-monsoon	0.995	95.875	7.2	0.960	90.566	11.3
Winter	0.993	95.872	0.8	0.965	90.524	0.7
Summer	0.991	95.869	3.5	0.962	90.549	5.5
Annual	0.993	95.869	38.0	0.962	90.580	58.5

Table 5. Values of MPIs given by Gumbel and Frechet distributions for seasonal and annual rainfall patterns of Godavari river basin

Rainfall series	Values of MPIs					
	Gumbel			Frechet		
	CC	MEF (%)	RMSE (mm)	CC	MEF (%)	RMSE (mm)
Monsoon	0.982	86.930	63.0	0.952	83.549	71.0
Post-monsoon	0.982	86.929	6.6	0.950	83.539	7.7
Winter	0.985	86.980	1.7	0.955	83.584	1.8
Summer	0.980	86.910	3.5	0.950	83.532	4.0
Annual	0.982	86.931	74.8	0.952	83.548	84.5

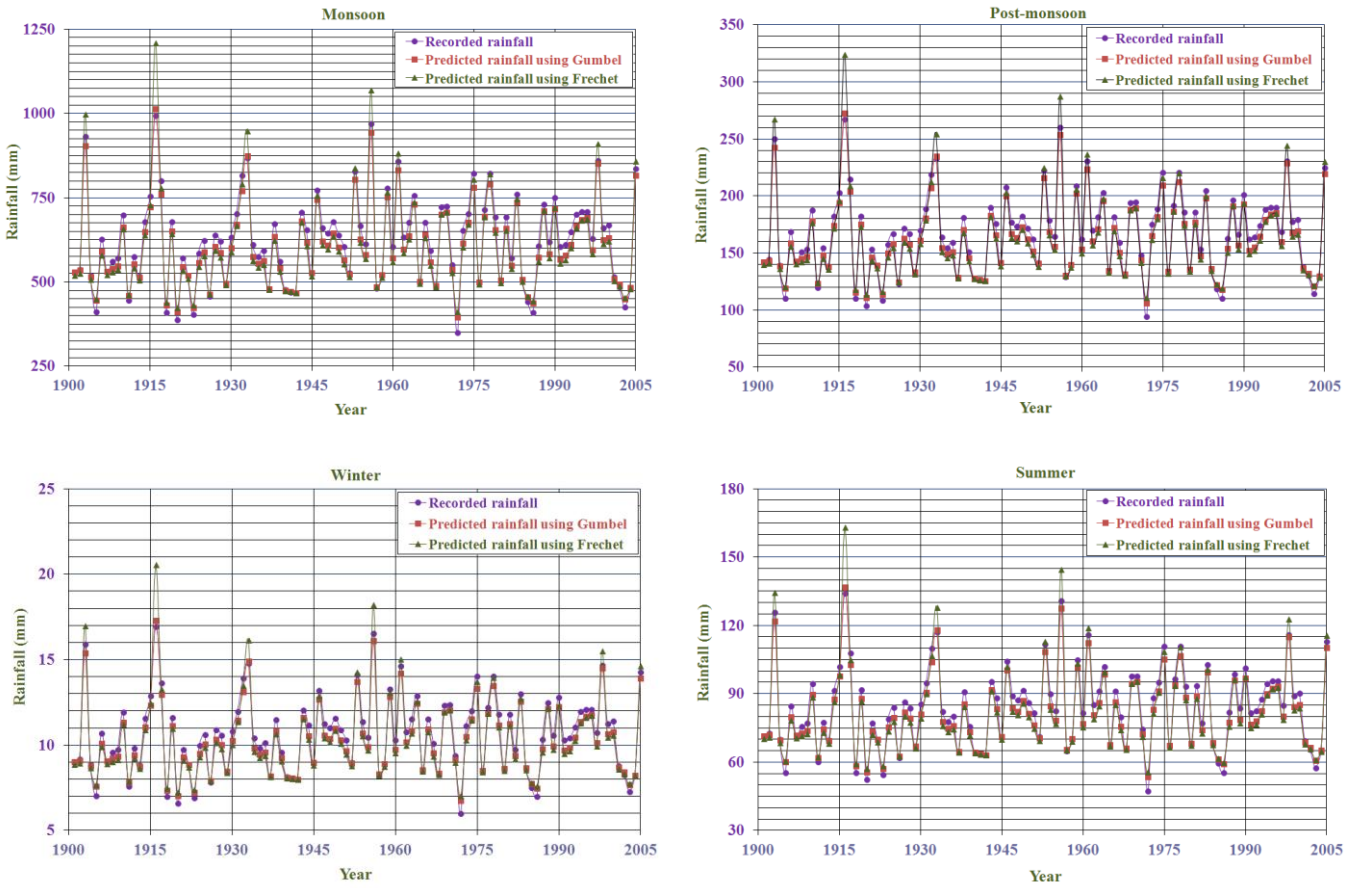

Figure 1. Plot of recorded and predicted seasonal rainfall using Gumbel and Frechet distributions for Krishna river basin

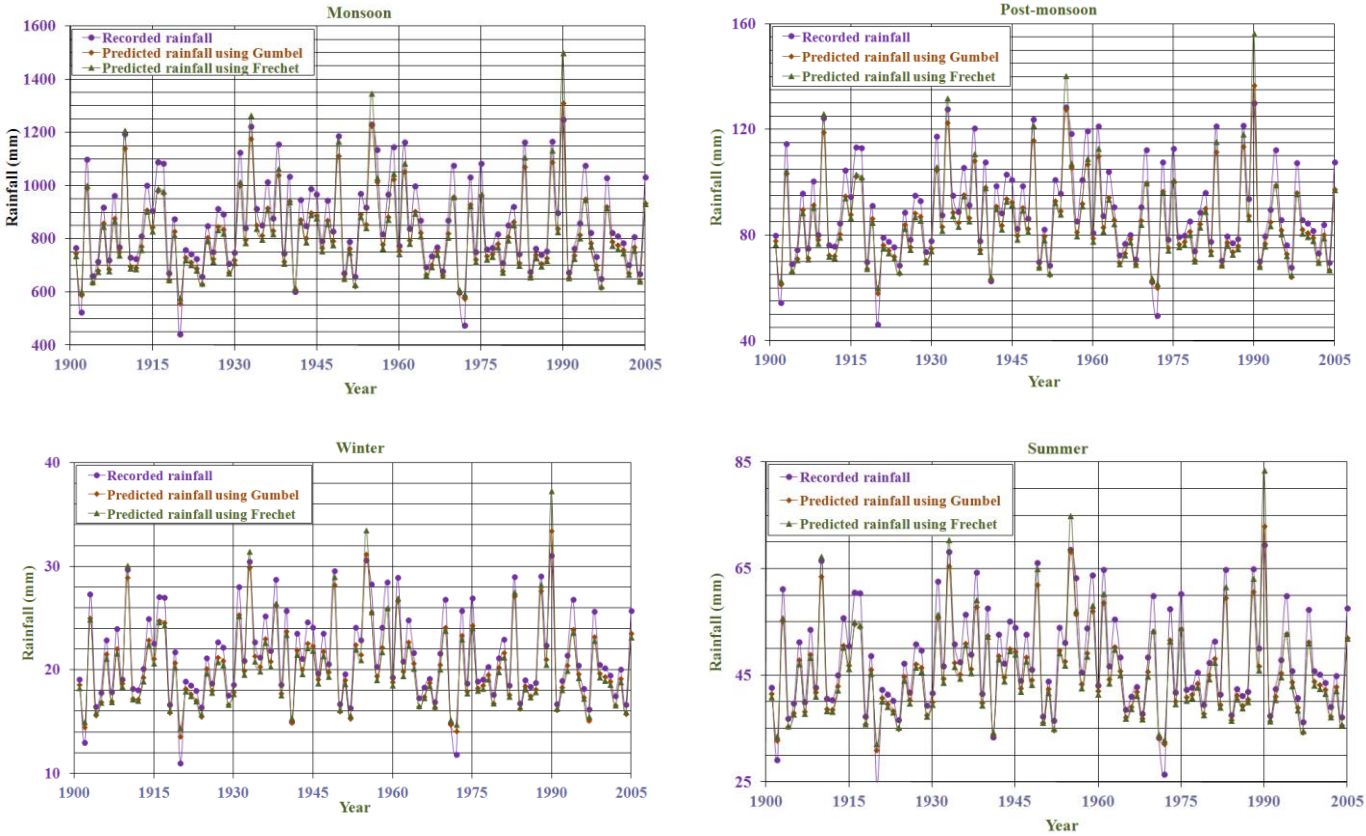

Figure 2. Plot of recorded and predicted seasonal rainfall using Gumbel and Frechet distributions for Godavari river basin

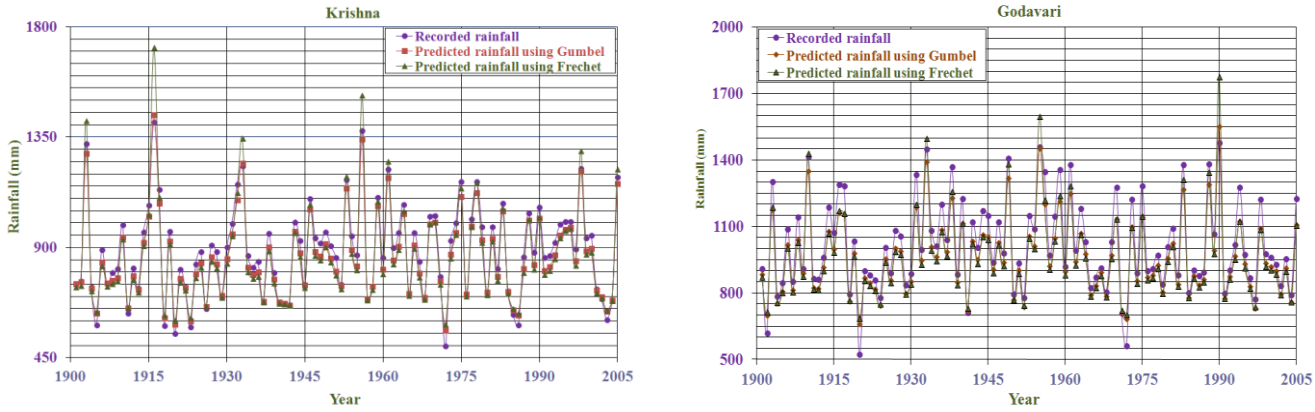

Figure 3. Plot of recorded and predicted annual rainfall using Gumbel and Frechet distributions for Krishna and Godavari river basins

From Tables 4 and 5, it may be noted that: (i) There is generally good correlation between the recorded and predicted rainfall using Gumbel and Frechet distributions, with the CC values varying from 0.950 to 0.995; (ii) For Krishna basin, MEF on prediction of seasonal and annual rainfall using Gumbel and Frechet are computed as 96% and 91% respectively; (iii) For Godavari basin, MEF on the predicted seasonal and annual rainfall using Gumbel and Frechet are noted to be about 87% and 84% respectively; (iv) The results indicated the RMSE on the predicted seasonal and annual rainfall given by Gumbel is minimum when compared with the corresponding values of Frechet for Krishna and Godavari; and (v) By considering the amount of variation in magnitude of MPIs, the study suggested the Gumbel distribution (using OSA) is better suited for prediction of seasonal and annual rainfall for Krishna and Godavari basins.

CONCLUSIONS

The paper described the procedures involved in prediction of rainfall using Gumbel and Frechet distributions for Krishna and Godavari river basins. The results of performance analysis described that the Gumbel distribution is better suited for prediction of seasonal and annual rainfall for the river basins under study. The results showed that the RMSE on the predicted rainfall using Gumbel, with reference to recorded rainfall, vary between about 1 mm to 27 mm for seasonal pattern; and 38 mm for annual pattern for Krishna basin. The results also showed that the RMSE on the predicted rainfall given by Gumbel distribution is about 75 mm for annual pattern and vary between about 2 mm to 63 mm for seasonal pattern for Godavari basin. The paper presented that the MEF in prediction of seasonal and annual rainfall using Gumbel distribution for Krishna and Godavari river basins are varying from 84% to 96%. The paper also presented that the CC in prediction of seasonal and annual rainfall using Gumbel for Krishna and Godavari river basins are varying from 0.980 to 0.995. The study showed that the results presented in the paper would be beneficial to the stakeholders while planning of irrigation and drainage systems as also for command area development in Krishna and Godavari river basins.

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Competing interests

The author declares that he has no competing interests.

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