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Assessment of Flow Discharge Prediction in Main Channels using GEP and Traditional Models

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ABSTRACT: Making accurate calculation of flood flow discharges has specific priority for many river engineering projects, flood control measures, and sediment transport problems. Nowadays, under the form of compound open channels, rivers have been widely used as flood conveyance systems for urban water management. Due to momentum transfer between main channel and flood plains, the flow hydraulic in compound channels is more complicate than the simple channels. Most studies in this field are focused on prediction of the total flow discharge in compound open channels. However, in flood conditions and in the case of spill of water on the flood plains, the bed and specially suspended sediment loads are mainly transported by the main channel flow discharge. In this study, using laboratory and field stage-discharge datasets from channels with compound sections, the individual flow discharge of the main channel is predicted applying gene-expression programming (GEP) then compared with traditional divided channel methods. Results showed that the proposed soft computing method with mean error of 8.2% has promising performance in prediction of subsection flow discharges for main channel. Furthermore, among the traditional methods, the diagonal (inclined) and vertical divided channel methods with mean errors of 10.6 and 18.2 % have greatest and lowest accuracies in estimation of main channel discharge, respectively.

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INTRODUCTION

Since ancient time, it has been a human tendency to settle within the close proximity to the river system for accomplishing their most basic needs along with their entertainment and transportation (Devi et al., 2016). At extreme discharge conditions floods may occur in the rivers that could severely damage nearby infrastructure and also cause casualties (Huthoff, 2007). Over more than three decades, hydraulics of compound channels has been extensively investigated. It is found that flow hydraulic characteristics are completely different in main channel and adjacent floodplains, and hence, a strong momentum exchange takes place. In this condition, it's necessary to treat the compound channel into subsections for flow velocity and discharge computation (Lambert and Myers, 1998). For dividing of compound channels, there are many approaches among them are vertical, horizontal and diagonal planes. Currently, for hydraulic modeling of river compound channels (e.g. water surface profile computations, bed shear stress determination across the river, and sediment transport modeling) the vertical dividing method has widespread applications in one-dimensional commercial mathematical packages such as MIKE11, HEC-RAS, ISIS and SOBEC (Huthoff et al., 2008). However, this method suffers from great over-prediction error for discharge estimation in the case of river and flume compound sections (Martin and Myers, 1991; Ackers, 1992).

In Fig. 1, a river compound open channel has been divided into three common cases of vertical, diagonal and horizontal planes. The associated important hydraulic and geometric parameters are also shown in this Figure 1.

In all these simple methods, it is assumed that there is no shear stress at the interface between main channel and floodplains (Chow, 1959). Experimental data obtained from large and small scale compound channels, have revealed that this assumption isn't correct and therefore, these methods especially the vertical divided approach are maybe very erroneous (Myers, 1987). From Figure 2, it can be clearly seen that due to strong apparent shear stress at the interface plane, a fully turbulent flow with high momentum transfer as well as strong longitudinal vortexes is induced.

For improvement and deal with large error of traditional divided channel methods, many modified approaches have been provided by several researchers (Wormleaton and Merrett, 1990; Ackers, 1992; Bousmar and Zech, 1999; Atabay and Knight, 2006; Huthoff et al., 2008; Yang et al., 2014; Kordi et al., 2015). In most of these studies, the main aim was precise prediction of total flow discharge or cross-sectional flow velocity. However, in many practical situations, distribution of flow rates in the main channel and over floodplains are more important. Ackers (1992) states that any suitable method should have such abilities to predict flow discharge in subsections, especially in the main channel, with sufficient accuracy.

In overbank flows, the floodplains convey considerable amount of flow with quite low magnitudes of velocity. Hence, in this condition, river system not only behaves as a conveyance but also as a storage or pond. It is recognized that for bed load and suspended load sediment transport, only the flow discharge in main channel is effective and floodplain's discharge has a negligible impact upon the subject. In fact the floodplains, due to their high capacities, play an important role in flood water level reduction, water retention and fine sediment deposition. These features are essential for wetlands restoration and preserve of river ecology as well as for success of flood mitigation works (Zahiri et. al., 2016).

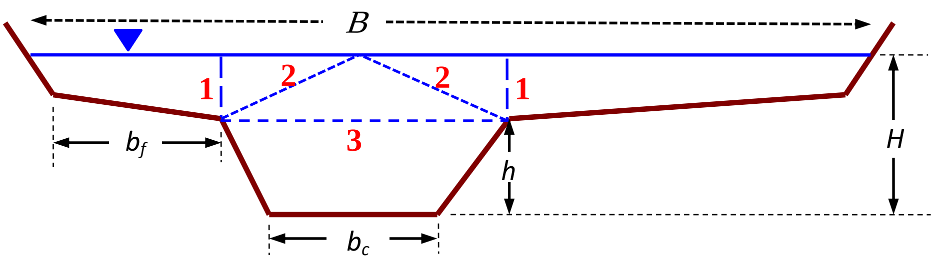


Figure 1. A compound river channel with common dividing channel methods, 1: vertical, 2: diagonal and 3: horizontal dividing models

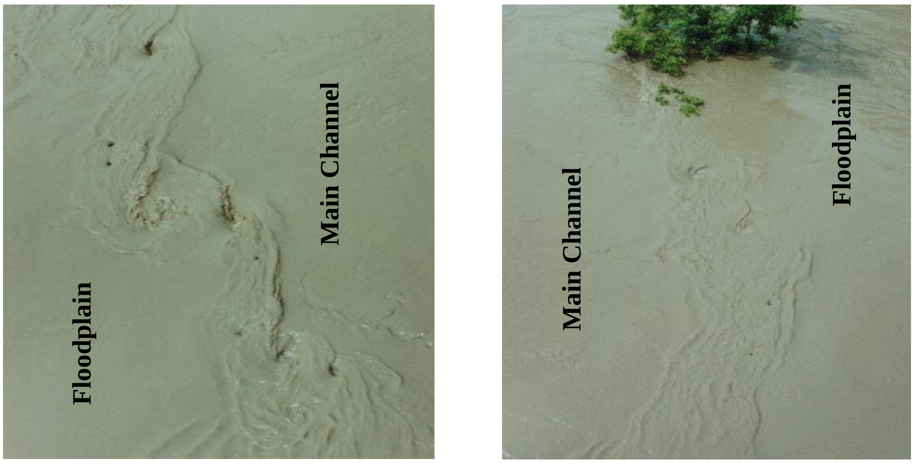


Figure 2. Momentum transfer and strong longitudinal vortexes at the main channel/floodplain interface in a compound river channel during flood condition

In sediment transport point of view, the bank-full level of a river is defined as the level at which the water has its maximum power and energy to move suspended sediment. In river compound channels and at the flood conditions, the water rises above the bank-full level and flow spills over the floodplains. In this case, the average flow velocity and more importantly the stream power dramatically reduces. By considerable reduction of the stream power, the sediment transport capacity is reduced as well (Tang and Knight, 2006). Thus, for better monitoring of river behavior during flood events, accurate computation of flow velocity and hence sediment transport capacity of both main channel and floodplains are needed. It should be noted that for computation of sediment transport capacity in flooded rivers, the individual flow discharge of main channel should initially be separated from total flow rate of the river. It then should be put into a suitable empirical sediment transport equation.

Nowadays, many artificial intelligence models have been presented and used for solution of various problems in hydrology and hydraulic sciences. For instance, in the case of an important subject such as flow discharge prediction in compound channels, many approaches including artificial neural networks (ANNs) (Zahiri and Dehghani, 2009; Parsaie et al., 2016), linear genetic programming (LGP) (Azamathulla and Zahiri, 2012), model tree (MT) (Zahiri and Azamathulla, 2014), and group method of data handling (GMDH) network (Najafzadeh and Zahiri, 2015) were applied to improve the traditional models of vertical divided channel method. All these works place considerable emphasis on total flow rate prediction and the sub-section's flow discharges of main channel and floodplains have received less attention.

Among artificial intelligence approaches, Gene-Expression Programming (GEP) model extracted an input-output model based best formulations to characterize physical meaning of governing parameters in hydrologic problems (Zahiri et al., 2016). GEP methodology was implemented for wide ranges of different problems in hydrologic engineering. A large number of investigations were conducted by the GEP approach for natural processes, as prediction of suspended sediment concentration (Aytek and Kisi, 2008; Kisi and Guven, 2010; Zakaria et al., 2010; Azamathulla et al., 2011; Kisi et al., 2012; Roushangar et al., 2014), evaluation of daily evapotranspiration (Guven et al., 2007; Shriri et al., 2014), development of stage-discharge curve (Guven and Aytek, 2009), rainfall prediction (Kashid and Maity, 2012), and prediction of dispersion coefficient in rivers (Sattar and Gharabaghi, 2015).

MATERIAL AND METHODS

Theory of Traditional Divided Channel Methods

In Fig. 1, main channel and floodplains sections separated by three dividing methods (e.g., vertical, diagonal, and horizontal) are shown. Total flow discharge is the sum of discharges calculated separately in each subsection using an appropriate conventional friction formula, for example, Manning's equation (Chow, 1959):

$$Q_{DCM} = \sum_{i=1}^{3} Q_i = \sum_{i=1}^{3} \frac{A_i R_i^{2/3} S_0^{1/2}}{n_i} \qquad (1)$$

Where Q_{DCM}, Q, R, n, A, and S_0 are total flow discharge in compound channel, subsection flow discharge in compound channel for main channel or floodplains, hydraulic radius of cross-section, Manning roughness coefficient, area, and longitudinal slope of channel, respectively. Also, in Eq.(1), i refers to each subsection (main channel or floodplains).

Data Processing

In this study, 147 laboratory stage-discharge datasets were collected from 14 different compound channel sections. These data are related to the main channel flow discharges. Out of datasets, 75% of them are used to train datasets and the remaining 25% is devoted to perform testing stage. The datasets are included those of bank-full depth, bed slope, and main channel and floodplain characteristics such as width, side slope, flow discharge, flow depth and Manning roughness coefficient. These datasets were collected form experimental works carried out by HR Wallingford (FCF) in compound channel flumes with large-scale facility (Knight and Sellin, 1987; Bousmar and Zech, 1999; Bousmar et al., 2004; and Fernandez et al., 2012). The ranges of geometric and hydraulic characteristics of compound channels used in this study are listed in Table 1.

Table 1. Overview of experimental data sets used for development and assessment of GEP model

Variable definition	Variable range	Mean value
Bank-full height, h (m)	0.05-0.2	0.103
Flow depth, H (m)	0.058-0.32	0.1482
Main channel width, b_c(m)	0.05-1.6	0.89
Floodplain width, b_f(m)	0.16-6	1.49
Bank-full discharge, Q_b (m^3/s)	0.0023-0.2162	0.096
Total flow discharge, Q_t (m^3/s)	0.003-1.1142	0.2145
Main channel flow discharge, Q_{mc} (m^3/s)	0.00233-0.6271	0.1499
Bed slope (S_0)	0.00099-0.013	0.0021

For model development, it is assumed, somewhat similar to Ackers (1992) approach, that subsection of flow discharges are dependent on three input dimensionless parameters including relative flow depth (floodplain depth to main channel depth, *Dr*), coherence parameter, and calculated flow discharge using vertical divided channel method. Accordingly, the following functions are proposed to predict the flow discharge both in main channel and floodplain, respectively:

$$Q_{mc} = f\left(Dr, COH, Q_{mc-VDCM}\right) \qquad (2)$$

where Q_{mc} is predicted flow discharge in main channel. Also, $Q_{mc-VDCM}$ is sub-section discharge obtained by vertical dividing method. *COH* is expressed as follows (Ackers, 1992):

$$COH = \frac{\left(1+A_*\right)^{1.5} \Big/ \sqrt{\left(1+P_*^{1.33} n_*^{\;2} \big/ A_*^{0.33}\right)}}{1 + A_*^{1.67} \big/ n_* P_*^{0.33}} \qquad (3)$$

where P is the wetted perimeter and * denotes the ratio of floodplain to main channel's value. The *COH* parameter defines the degree of interaction between main channel and floodplains. This value goes to unity for compound channels with least degree of interaction (e.g. smooth channels in the case of high flows) and goes to 0 for high degree of interaction effect (e.g. low flows in rough channels).

Development of GEP Model

Recently a new technique called GEP was developed which is an extension of the GP approach. The GEP is a search model that evolves computer programs in forms of mathematical expressions, decision trees, and logical expressions (Azamathulla and Haque, 2012; Ferreria, 2006). In addition, the GEP model has attracted the attention of investigators in prediction of characterizations in hydraulic problems. This research represents GEP model for evaluation of prediction of flow discharge in main channel. The GEP approach is coded in forms of linear chromosomes, which are then expressed into Expression Trees (ETs).

In fact, the ETs are sophisticated computer programming which are usually evolved to solve a practical problem, and are selected accordingly to their fitness at solving that problem. The corresponding empirical expressions can be obtained from these trees structures. A population of the ETs will discover traits, and therefore will adapt to the particular problem they are employed to solve (Azamathulla and Haque, 2012; Ferreria, 2006).

Development of the GEP approach includes five steps. The first step is to select the fitness function, fi, of an individual program (i). This item is evaluated as follows:

$$f_i = \sum_{j=1}^{C_t} (M - |C_{(i,j)} - T_j|) \qquad (4)$$

in which M, $C_{(i,j)}$ and T_j are the selection range, value returned by the individual chromosome i for fitness case j and the largest value for fitness case j, respectively.

In the second stage, the set of terminals T and the set of function F were selected to generate the chromosomes. In this study for main channel discharge, the terminals include three independent parameters in form of $T(Q_{mc}) = (Dr, COH, Q_{mc-VDCM})$.

To find the appropriate function set, it is necessary to peer review previous investigations in this area. In this way, four basic operators (+,-,*,/) and basic mathematical functions (√, power, sin, cos, exp) were applied to predict the flow discharge modeling. The third step is to configure the chromosomal architecture. The fourth step is selection of liking function. Finally, for the fifth stage, the sets of genetic operators and their rate are selected. The other details related to the architecture of the GEP modeling were expressed in the literature (Azamathulla and Haque, 2012). In this study, characterizations of the flow discharge in form of Q_{mc} are predicted using the GEP model.

Furthermore, the functional set and the operational parameters applied in the proposed GEP model are presented in Table 3. The best formulations of GEP models for evaluation of the flow discharge, as a function of *Dr*, *COH* and vertical divided discharge, are obtained as following:

$$Q_{mc} = \frac{Q_{mc-VDCM}}{\sqrt{Dr} - 8.181} + \frac{e^{COH} + Q_{mc-VDCM} - e^{Dr}}{-5.103^3} + Q_{mc-VDCM} \qquad (5)$$

Table 3. Parameters of the optimized GEP model

Parameter	Description of parameter	Setting of parameter
p_1	Function set	+, -, ×, /
p_2	Population size	250
p_3	Mutation frequency %	96
p_4	Crossover frequency %	50
p_5	Number of replication	10
p_6	Block mutation rate %	30
p_7	Instruction mutation rate %	30
p_8	Instruction data mutation rate %	40
p_9	Homologous crossover %	95
p_{10}	Program size	initial 64, maximum 256

Statistical Measures for Models Evaluation

To evaluate the accuracy of the explicit equations extracted by the GEP, MT, EPR, and traditional models in both training and testing phases, some common statistical measures including correlation coefficient (R), the mean squared error (MSE), the mean absolute percentage of error (MAPE) and performance index (ρ), are used as follows:

$$R = \frac{\sum_{i=1}^{N} x_i y_i}{\sqrt{\sum_{i=1}^{N} x_i^2 \sum_{i=1}^{N} y_i^2}} \qquad (6)$$

$$MSE = \frac{\sum_{i=1}^{N} (X_i - Y_i)^2}{N} \qquad (7)$$

$$MAPE = \frac{\sum_{i=1}^{N} \frac{|X_i - Y_i|}{X_i}}{N} \times 100 \qquad (8)$$

$$\rho = \frac{\sqrt{MSE}}{\overline{X}} \frac{1}{1+R} \qquad (9)$$

where $x_i = (X_i - \overline{X})$, $y_i = (Y_i - \overline{Y})$, $\overline{X}$ is the average of X (measured outputs), $\overline{Y}$ is the average of Y (predicted outputs) and N is the data point's number for proposed models evaluation (experimental data). The last statistical parameter (ρ) is a new criterion proposed by Gandomi and Roke (2013) which combines both correlation and error functions. This is a robust statistical measure of model performance based on combined impacts of RMSE and R. Similar to other error functions, lower values of ρ indicated better fit.

RESULTS AND DISCUSSION

In Figure 3 results of three types of divided channel methods are shown for flow discharges in main channel. As seen, vertical and horizontal approaches have over and under predictions, respectively. The over-prediction of the vertical divided method is due to the interaction effect that causes the actual discharge to decreases in the main channel and increases in the floodplains. Errors of these two methods are growing with increasing flow discharges, especially for vertical method. Among these approaches, the method of diagonal planes has a suitable result, for both small and large main channel discharges. From practical point of view, based on this figure, it is revealed that the vertical approach has produced the biggest error for calculation of discharge in the main channel.

Scatter plots for performances of results for training of the GEP model in main channel is given in Figs. 4(a) and (b) for all data and test data, respectively. As can be seen, the proposed model has produced enhanced results, in total ranges of the main channels' flow discharges. For better analysis, results of the proposed model for flow discharge prediction in main channel for testing phase are given in Table 2. In this table, the statistical results of traditional divided channel approaches are also presented. From this table, it can be concluded that according to the error functions, especially the mean absolute percentage of error (MAPE), the proposed model give better results than the traditional approaches. As can be seen, quantitatively statistical parameters obtained in the training stage indicated that GEP model predicts the flow discharge for main channel with considerably higher accuracy GEP (MSE=0.00026, MAPE=8.21%, and ρ=0.054) than those obtained using traditional models. Furthermore, among the traditional methods, the vertical approach, which is currently used in many engineering packages, with mean absolute error of 19%, has the lowest accuracy, while the diagonal approach is the best one. Diagonal dividing model indicated more efficient performance in terms of MSE (0.0004) and ρ (0.058) for the prediction of the main channel flow discharges than those computed by vertical (MSE=0.0026 and ρ=0.152) and horizontal (MSE=0.0015 and ρ=0.117) dividing techniques.

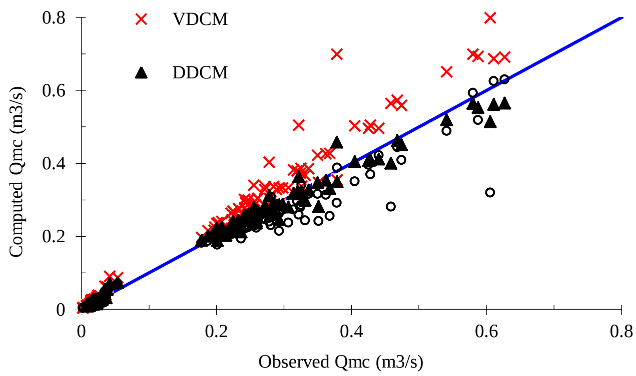
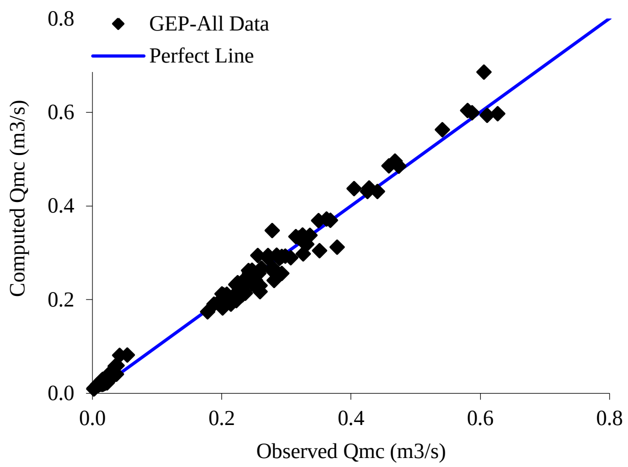
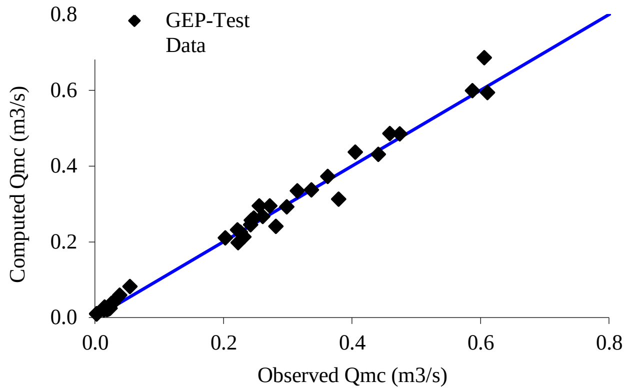


Figure 3. Flow discharge calculation of traditional divided channel methods for main channel



(a)



(b)

Figure 4. Scatter plot for flow discharge calculation in main channel for (a) all data and (b) test data

Table 2. Correlation and error measures for testing phase of GEP model and different traditional predictors (divided channel methods) for flow discharge in main channel

Model	R	MSE	MAPE (%)	ρ
GEP	0.995	0.00026	8.21	0.054
Vertical Divided Method	0.986	0.00265	18.20	0.152
Diagonal Divided Method	0.994	0.00044	10.55	0.058
Horizontal Divided Method	0.980	0.00153	13.20	0.117

The testing procedure of the GEP new formulae is may be also a concern. In this study, although the proposed formulae is dimensional-independent, a more reliable testing was performed by using a real life river geometry not used for GEP training. There are very few suitable field data in the case of compound river channels. Among them, River Severn at Montford bridge (Ackers, 1991; Mc Gahey et al., 2006). The geometric characteristics of this river section are explained in many researches. The River Severn in Montford Bridge has an asymmetric section with two inclined berms being 63m and 21m wide, respectively. The Manning's roughness of the main channel is 0.03, and for left and right floodplains are 0.028 and 0.04, respectively (Knight et al. 1989). The main channel width and height are 17m and 6m, and the side slopes are 1.5. The bed slope of the river is 0.000185.

In Figure 5, the accuracy of GEP model results are tested against the traditional divided channel methods for prediction of main channel flow discharge. As can be seen, although the number of data points are too few, but the GEP model indicates very interesting accuracy in comparison with all traditional divided channel methods. The mean error of the GEP results is nearly 0.7%, while the error magnitudes for VDCM, HDCM, and DDCM are 16.7%, 8.5%, and 8.8%, respectively. This testing example clearly shows the suitable applicability of the new GEP formulae for prediction of main channel flow discharges in the case of compound river channels. However, with the lack of river data, further researches and extra suitable field data are needed for complete validation of proposed model.

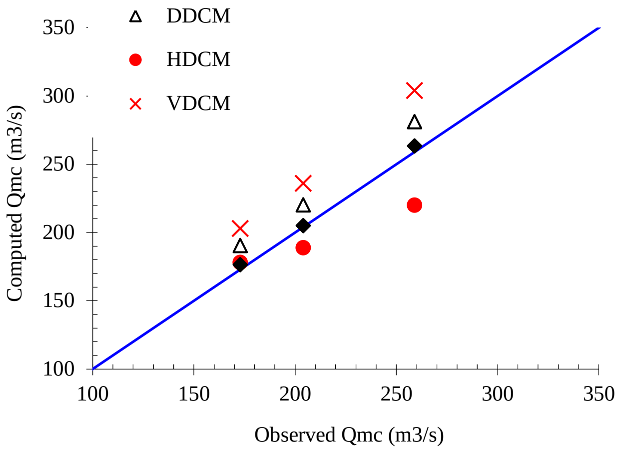


Figure 5. Predictions of the traditional divided channel methods and GEP model for main channel flow discharge (River Severn at Montford Bridge)

CONCLUSION

In the present investigation, a new application of GEP model into a fundamental hydraulic engineering problem has been presented. Through this study, based on the training and testing stages, a best explicit equation for main channel flow discharge was developed using GEP model. In case of traditional models, vertical, diagonal and horizontal divided channel methods were applied to evaluate the flow discharge for compound open channels with different hydraulic and geometric conditions. Statistical computations conducted in this study indicated that the proposed approach has provided better predictions of the main channel flow discharge, compared to the traditional dividing techniques. In the case of traditional models, vertical and horizontal approaches have over and under predictions, respectively.

Among these approaches, the diagonal dividing approach provided relatively more accurate prediction of flow discharge (MSE=0.00044, MAPE=10.55%, and ρ=0.058) in comparison with vertical (MSE=0.00265, MAPE=18.2%, and ρ=0.152) and horizontal (MSE=0.00153, MAPE=13.2%, and ρ=0.117). However, it is found that GEP model has predicted the flow discharge in main channel with more reliable accuracy in term of MAPE (8.21%).

In case of practical applications, most concerns of issue are founded in highlights of physical meaning of performances for main channel. Errors of traditional methods are very large and grow with increasing flow discharges, especially for vertical divided method. On the other hands, the diagonal dividing method provides relatively suitable results, although the errors of computations increase for large main channel's discharges.

Competing interests

The authors declare that they have no competing interests.

REFERENCES

Ackers, P. (1992). Hydraulic design of two-stage channels. Journal of Water and Maritime Engineering. 96: 247-257.

Atabay, S., and Knight, D.W. (2006). 1-D modelling of conveyance, boundary shear and sediment transport in overbank flow. J. Hydraul. Res., IAHR. 44(6): 739-754.

Aytek A, Kisi O., (2008). A genetic programming approach to suspended sediment modeling. J Hydrol. 351: 288-98.

Azamathulla, H. M. and Haque, A. A. M. (2012). Prediction of Scour Depth at Culvert Outlets Using Gene-Expression Programming. International Journal of Innovative Computing, Information and Control, 8: 5045-5054.

Azamathulla, H.Md., AbGhani, A., Leow, C.S., Chang, C.K., and Zakaria, N.A. (2011). Gene-expression programming for the development of a stage-discharge curve of the Pahang River. Water Resources Management. 25(11): 2901-2916.

Azamathulla, H.Md., and Zahiri, A. (2012). Flow discharge prediction in compound channels using linear genetic programming. J. Hydrol. Ser. C, 454: 203-207.

Bousmar, D. and Zech, Y. (1999). Momentum transfer for practical flow computation in compound channels. J. Hydraul. Eng., ASCE. 125(7): 696-70.

Bousmar, D., Wilkin, N., Jacquemart, H. and Zech, Y. (2004). Overbank flow in symmetrically narrowing floodplains. J. Hydraul. Eng., ASCE. 130(4): 305-312.

Chow, V.T. (1959). Open channel hydraulics, McGraw-Hill, London.

Devi, K., Khatua, K.K., and Das, B.S. (2016) Apparent shear in an asymmetric compound channel. River Flow 2016: Iowa City, USA.

Fernandes, J.N., Leal, J.B. and Cardoso, A.H. (2012). Analysis of flow characteristics in a compound channel: comparison between experimental data and 1-D numerical simulations. Proceedings of the 10th Urban Environment Symposium, 19: 249–262.

Ferreria, C. (2006). Gene-expression programming", Mathematical Modeling by an Artificial Intelligence, Spriner, Berling, Heidelberg, New York, 21.

Gandomi, A.H., and Roke, D.A. (2013). Intelligent formulation of structural engineering systems. In: Seventh M.I.T. Conference on Computational Fluid and Solid Mechanics, Massachusetts Institute of Technology, Cambridge, MA.

Guven A, Aytek A, Yuce MI, Aksoy H. (2007). Genetic programming-based empirical model for daily reference evapotranspiration estimation. Clean -Soil Air Water. 36: 905-912.

Guven A, Aytek A. 2009. New approach for stage–discharge relationship: gene-expression programming. Journal of Hydrologic Engineering.14(8):812-820.

Huthoff, F. 2007. Modeling hydraulic resistance of floodplain vegetation. PhD Thesis, Twente University, the Netherland.

Huthoff, F., Roose, P.C., Augustijn, D.C.M., Hulscher, S.J.M.H. (2008). Interacting divided channel method for compound channel flow. J. Hydraul. Eng., ASCE. 134(8), 1158-1165.

Kashid, S.S., Maity, R., (2012). Prediction of monthly rainfall on homogeneous monsoon regions of India based on large scale circulation patterns using Genetic Programming. Journal of Hydrology. 454-455: 26-41.

Kisi, O., and Guven, A., (2010). A machine code-based genetic programming for suspended sediment concentration estimation. Advances in Engineering Software. 41: 939-945.

Kisi, O., Hosseinzadeh Dalir, A., Climen, M., Shiri, J., (2012). Suspended sediment modeling using genetic programming and soft computing techniques. Journal of Hydrology. 450-451: 48-58.

Knight, D.W. and Sellin, R.H.J. (1987). The SERC flood channel facility. Journal of Institution of Water and Environment Management. 1(2): 198-204.

Lambert, M.F. and Myers, R.C. (1998). Estimating the discharge capacity in straight compound channels. Water, Maritime and Energy. 130, 84-94.

Martin, L.A. and Myers, R.C. (1991). Measurement of overbank flow in a compound river channel. Journal of Institution of Water and Environment Management, 645-657.

Myers, W.R.C. (1987). Momentum transfer in a compound channel. Journal of Hydraulic Research, 16: 139-150.

Najafzadeh, M., and Zahiri, A. (2015). Neuro-Fuzzy GMDH Based Evolutionary Algorithms to Predict Flow Discharge in Straight Compound Channels. J. Hydrol. Eng. ASCE. DOI: 10.1061/(ASCE)HE.1943-558.001185.

Roshangar, K., Mouaze, D., Shiri, J., (2014). Evaluation of genetic programming-based models for simulating friction factor in alluvial channels. Journal of Hydrology, 517: 1154-1161.

Sattar, A.M.A., and Gharabaghi, B., (2015). Gene expression models for prediction of longitudinal dispersion coefficient in streams. Journal of Hydrology. 524: 587-596.

Shiri, J., Sadraddini, A.A., Nazemi, A.H., Kisi, O., Landeras, G., Farokhi Fard, A., Marti, P., (2014). Generalizability of Gene Expression Programming-based approaches for estimating daily reference evapotranspiration in coastal stations of Iran. Journal of Hydrology. 508: 1-11.

Tang, X., and Knight, D.W. 2006. Sediment transport in river models with overbank flows. J. Hydraul. Eng., ASCE, 132(1): 77-86.

Wormleaton, P.R. and Merrett, D.J. (1990). An improved method of calculation for steady uniform flow in prismatic main channel/floodplain sections. J. Hydraul. Res., IAHR, 28: 157-174.

Yang, K., Liu, X., Cao, S., and Huang, E. (2014). Stage-discharge prediction in compound channels. J. Hydraul. Eng., ASCE, 140(4): 353-361.

Zahiri, A., and Azamathulla, H.Md. (2014). Comparison between linear genetic programming and M5 tree models to predict flow discharge in compound channels. Neural. Comput. Appl. 24(2): 413-420.

Zahiri, A. Dehghani, A.A. and Azamathulla, H.Md. "Chapter 4. Application of gene-expression programming in hydraulic engineering". Handbook of Genetic Programming Applications. A.H. Gandomi, A.H. Alavi and C. Ryan (eds), Springer, 71-98.

Zahiri, A., and Dehghani, A.A. (2009). Flow discharge determination in straight compound channels using ANN. World. Acad. Sci. Eng.Technol., 58:1-8.

Zakaria, N.A, Azamathulla, H.Md, Chang, C.K and AbGhani, A. (2010). Gene-Expression programming for total bed material load estimation-A Case Study. Science of the Total Environment, 408(21): 5078-5085.